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THE STORY OF DOMESTIC LIGHTING

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All living things are dominated by the light and warmth of the sun. Sunlight is so much the essence of living things that light has been much more to mankind than a physical phenomena. We have physical light for our seeing, symbolic light for our minds, and sacred light for our spirit. Indeed, light has been so interwoven with life and human welfare that a large volume would be required to follow all its influences.

At first the sun itself was regarded as an object of worship. Later religions incorporated lights in their rituals as symbols of the central source of all light. Ancient Greek mythology tells us how the light of the sun was brought to earth so that fire and its light might be employed to greater advantage in the service of man. Prometheus, gifted with great wisdom, ascended to the heavens, lighted a torch at the chariot of the sun and brought fire down to the earth. It is a pretty story and provides us with a starting point for the story of lighting.

Once man sought shelter indoors, in deep caves or habitations, to escape from enemies, rain, and cold, he had to find a means of admitting light during the day and, after the sun had gone down, he had to learn how to bring light into his shelter. It is this form of light, "the torch of civilization," with which we are concerned.

The earliest known forms of lighting are about 8000 years old and yet the history of the white man in America, which does not yet encompass five centuries, covers practically the whole story of the methods used in artificial lighting. The stagnation in improvement of such an important thing as indoor illumination is all the more striking when viewed against the background of progress invention made in other fields. For more than 7500 years there was no basic change in the

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design of the oil lamp. Modern lighting is actually only little more than one hundred years old.

It is interesting to note that, until about a century ago, all forms of indoor lighting were associated with a flame. From the very earliest time, man-made light had been produced by burning some substance and, this conception of light prevailed until the introduction of electric light.

The first man who drew a flaming brand from the fire and used it to provide light and protection beyond the range of the fire's glow, was

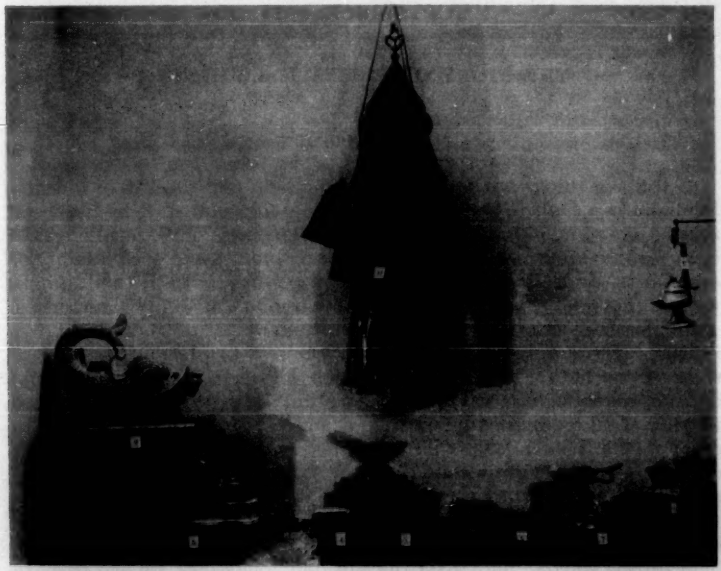

Photo by J. J. Barton

Some ancient lamps.

the inventor of the torch. Long after torches had come into use (and the candle and oil lamp for that matter) an open wood fire in an iron basket set on a pole remained a source of illumination for out of doors. The sports arenas and larger indoor feasts of the Greeks were lighted by piles of resinous woods soaked in fat burning in metal holders.

Appearing first as a simple brand drawn from the fire, the torch underwent a process of refinement and elaboration. Although we customarily think of it as borne in the hand and used for outdoor illumination, it was soon recognized that the torch did not have to be portable.

This led to the creation of the first lighting fixture. In its very crudest form the torch was stuck into the crevice between two stones, but the metal holders gradually evolved until we arrived at the ornate development of the cresset and flambeau.

We have no knowledge of man's early experiences with wood which would enable him to realize that resinous wood burned better than wood without resin. What we do know for certain is that the knowledge must have come to him at an early stage of his civilization, for some very ancient lamps have been discovered containing resin which had been extracted from the wood. These may even have been the forerunners of the oil lamp.

When the earliest settlers landed on the eastern shores of what are now the United States, they were, as far as lighting goes, only a step in advance of the aboriginal inhabitants. The domestic fire was the means of keeping at bay the deepest shadows of night. Except for the flames of their log fires, they had only brands of burning wood to light them in the darkness. These colonists brought with them a knowledge of improved lighting methods, such as candles and oil lamps, but their supplies were pitifully small and had to be used sparingly because there was no domestic means of replenishing supplies. The Indians made no use of candles and they knew nothing of the oils that could be burned in the lamps. Such illumination material as they employed was restricted to modifications of the torch.

Fortunately the pitch-pine was a common tree in America and furnished a plentiful supply of "candlewood," or lightwood as it was called in Virginia. From this tree could be made both the torch and the splinter. The torch was simply a good sized lump of blazing wood, suitable only for outdoor illumination, but the splinter could be stuck between the stones of the fireplace or held in an improvised holder and could be used indoors. Splinters were easily cut from a log of straight-grained wood without knots, and did not exceed the thickness of a candle. They found extensive use in many other parts of the world where resinous woods are to be found. Until comparatively recently they were still common in those parts of Russia where the magnificant pine forests furnished inexhaustible supplies of candlewood.

In order to ensure uninterrupted burning, splinters had to be held with the lighted tip downward. It must be confessed that they had their drawbacks even in an age when users were not so fastidious as they are now. One of the Puritan fathers (Winter) writing in 1642 says: "Out of these pines is gotten the candlewood that is so much spoken of, which may serve as a shift for the poorer folk, but I cannot commend it for singular good because it droppeth a pitch form of substance where it stands." Oddly enough, the suffocating smoke that accompanies the burning of pitchpine does not appear to have troubled

Winter, but the pitch he complained about was an admitted nuisance until the householder made a small pan to catch the drippings.

The pine splinter was popular among the poorer folk for the very good reason that candles cost eight cents each and, as money had a much higher purchasing power in those days, this was regarded as riotous extravagance. The splinter continued in use for many a long day after better lighting facilities had been evolved. Within living memory in New Orleans, negroes sold bundles of "lightwood" in the streets.

The native Indians used torches made from thin bundles of pitch-pine. This was probably the settlers introduction to the wood, since resinous woods are not common in Britain from whence the settlers had come. In place of splinters the British had been accustomed to using rushlights. As the new colonists spread out from the coastline some of them came to live near to swamps where the "cat-'o-nine-tails" type of rush grew abundantly. These immediately suggested the rushlights so familiar in England. An interesting account of the making of rushlights will be found in Gilbert White's delightful *Natural History of Selborne*. The rush was peeled except for a strip (sometimes two strips) which was left for support but which had the added effect of causing the burned material to twist to one side and so to fall off, thereby reducing the need for snuffing.

These rushlights provide us with what is probably the earliest evidence of American-made lighting apparatus, for they had to have some kind of a holder. To induce a wooden splinter to burn, the lighted end was turned downward, but a burning rushlight had to be held erect. Consequently, the colonists made holders of upright metal rods with an adjustable ring to hold the rushlight. The rush was held at a slight slant to encourage the ash to fall away unaided.

The light given off by a splinter or a rushlight was not much superior to that which we get today from a match. Mention of matches reminds us that care had to be taken in colonial days to keep a fire burning so that additional lights could be obtained. The Indians made fire by rubbing together two pieces of dry wood until enough heat was generated to cause the wood to burst into flame. The colonist had to follow this example if he did not have a tinder-box or burning glass at hand. The tinder-box contained a flint and steel to strike off sparks that would kindle a piece of half-charred linen, a few dried leaves, or a handful of wood chips. Kindling a fire was difficult enough to force everyone to go to some pains in order to keep a spark of fire glowing continuously. If the last ember was extinguished it was common practice to borrow fire from a neighbor rather than go to all the trouble of starting a new one.

Candlewood torches or splinters and rushlights were thus the first methods of interior lighting adopted in this country. The rushlight was the first to undergo the process of improvement when a progressive

colonist dipped his rushes in grease, obtained after cooking deer or bear flesh. This helped considerably to promote combustion, although it cannot have improved the odor accompanying the flame.

In the course of time enough grease became available for the more ambitious colonists to improve the lighting of their homes by making candles. Of course, candles were much older than this, having played a part in the domestic and religious life of all civilized people. Some of our calendars still record the feast of Candlemas (February 2nd) on which day, since the eleventh century, the church's candles for the ensuing year were blessed, and the popular preachers seized the oppor-

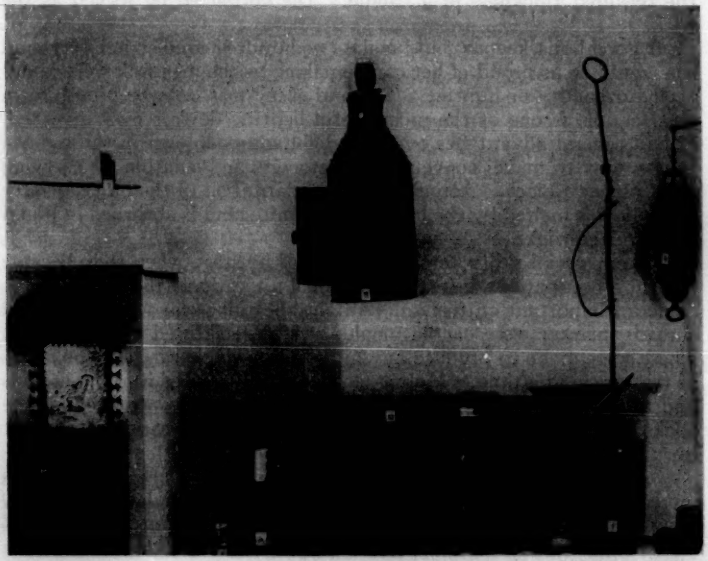

Photo by J. J. Barton

Splint, rushlight, and candle holders.

tunity of impressing their congregations by drawing lessons from the burning love of the Saviour. Candles used in the churches were made from beeswax, but domestic candles were made from tallow.

The colonists found a little beeswax, but by far the greater portion of their candles were made from animal fats. The manner of making them went unchanged for generations. All the early candles were "dips." The fat was boiled in water in a large pot. As the mixture boiled, the grease was skimmed off the surface and put into a separate receptacle. It was reboiled several times in order to remove as many

impurities as possible. Meanwhile, two slight poles were laid parallel over the backs of two chairs. A number of sticks with strings of twisted cotton dangling from them were laid across the two poles. As soon as the grease was ready, one of these sticks was taken up and held over the grease so that the cotton strings dipped into it; then it was removed, the stick laid back at rest across the two poles; and the hot grease that had collected on the strings was allowed to cool. By the time the last of the strings had been dipped the whole series was ready for another immersion in the same order. The process was repeated again and again, each dipping adding a fresh coat of grease until it had grown to the required thickness of a candle. The cotton string served as the wick.

A good housekeeper could make two hundred candles in a day, and she would be as proud of her candlemaking as the men folk were about their husbandry or hunting. She had every reason to be proud, for a good candle is one of the most useful lighting devices ever invented. It had several advantages over any light source known in those days. It was easy to make, convenient for storage, and suitable for carrying from place to place. Actually it is an adaptation of the oil lamps, for the tallow is melted by the heat when burning and is absorbed in liquid form by the wick.

When special skills were developed to the stage when they became full-time occupations instead of seasonal avocations, the tinsmith became an important craftsman. Among the things for which he found a ready market were candle moulds, which facilitated candlemaking. These were made to produce anything from a single candle to as many as two dozen at a time. Molten fat was simply poured into the moulds and allowed to cool. Special care had to be devoted to keeping the wicks taut and straight in the moulds, otherwise the candle would not burn equally.

As the population of the colonies grew and the demand for candles increased proportionately, men took to travelling from house to house, stopping a day or two in each place to make the winter's supply of candles. Where a sufficient number of people had congregated to form a town, the boiling of fat to make soap and candles for those who had neither the time nor the skill to make their own, became a recognized trade. Benjamin Franklin was brought up in the atmosphere of boiling fat, for his father was a soap and candle-maker.

Although a candle will burn efficiently without any special holder, there are too many occasions when a holder of some sort becomes more or less a necessity. Candle holders are, therefore, as old as candles themselves. In Roman days the holders had round sockets but for some reason not easy to understand these fell out of use. The earliest form of holder used in America was of the type which gave rise to the name of "candle-stick." It was no more than a pointed wooden stake

on which the candle was impaled. When the colonists found time to fashion better holders by moulding them out of clay, or hammering them from bog-iron, they produced what are called "prickets." These retained the sharp points which were driven into the candles.

However, as craftsmanship developed, there followed a variety of candle holders, with reversions to the socket type which fitted like a collar around the base of the candle. All kinds of material were tried—earthenware, glass, iron, tin, pewter, brass, silver, etc.—and many pleasing results were obtained. But while the artistic qualities of the holder were improving with every generation, a long time was to elapse before there was any improvement in the candle itself.

While thinking about candlesticks we should not overlook the lantern, which is not a distinct form of light but only a convenient means of carrying a light out of doors by protecting its flame from wind and weather. Candles were the almost universal lights of lanterns for many centuries, so that it is not surprising to learn that the cover or casing is nearly as old as the candle itself. Lanterns were usually constructed of four types: with simple perforations in a metal casing; in skeleton frames fitted with thin plates of horn; with a covering of paper or silk (as in Persia, Egypt, and China); fitted with glass sides, and the more or less modern types which are endless in their variety. Obviously, the first ones, with perforated sides, were of small utility as the metal casing obstructed most of the light rays. They were, however, the common barn light in Pennsylvania until well on into the nineteenth century. Called "lutzer" by the country folk of German extraction, they were often carried under the coat when exposed to the wind, otherwise out went the candle. Folding lanterns with mica windows were issued to United States forces during World War I.

Originally made of deer or bear fat in America, candles came to be made from equal parts of mutton and beef fat. They remained smelly and smoky. While the beeswax candle was not unknown, it never became common in this country, certainly never as common as it was in Europe. A peculiarly American wax candle was made from wax squeezed from the bay-berry. This was regarded as especially desirable because its fragrance in burning was in pleasant contrast to the foul odor of burning tallow. Because of the time required in making them and the small amount of wax obtainable, bay-berry candles were rare, and were reserved for special and ceremonious occasions. A large amount of berries was required to make a single candle and the wax obtained had to undergo repeated boilings and skimmings before it reached the delicate transparent green color at which stage it was ready for the dipping process.

Mutton and beef fat remained the principal materials from which candles were made until someone discovered the superior qualities of whale oil as an illuminant in lamps. Experiment with this revealed

that excellent candles could be made from the waxy substance found in the head of the sperm whale. From this the famous spermaceti candles were made. Candles remained at that stage until, in the early nineteenth century, a way was found to remove the glycerine (which was the cause of most smoke and smell) leaving only stearine of the animal tallow. This burned with a smokeless flame. About the same time, tapers and night-lights made their appearance, the latter in nurseries and sick rooms. The latter were tiny candles set on wooden floats placed in a bowl of water. When the candle burned down to the wood its flame was extinguished by the water.

Nor should it be forgotten that candles solved the first problems in street lighting in this country. In the old days in the large European cities people were wise to stay at home after dark to avoid the footpads and robbers who roamed the street, but if business or pleasure required that they should be abroad at night they hired a link-boy whose duty was to walk ahead carrying a lighted torch. These link-boys lingered on in the world's great cities almost to the time of the electric light, for other forms of street lighting had not proved satisfactory. As the first towns began to take form in this country, efforts were made to encourage the citizens to perform their individual share of lighting the streets by placing candles in their windows. The city of New York may not always have dazzled by its brilliance, but it tried hard to make its streets safe. In 1679 a law was passed ordaining that a light be placed on the end of a pole at every seventh house! And the light was no more than a candle in a lantern.

A writer in the *Domestic Encyclopaedia* published in Philadelphia in 1804, says that dipped candles were even at that time more common than those made in moulds. The same author declares that candles are preferable to lamps because they are less injurious to the eyes and lungs. There was probably some truth in that statement for the early lamps were disagreeable affairs.

Oil lamps are of very ancient origin. One made from sun-dried clay was dug from the ruins of the Babylonian city of Nippur, destroyed 6000 years before the Christian era, and there is no means of knowing how long oil lamps had been in use before that time. Herodotus describes a procession of lamps in Egypt in the year 445 B.C., and says the number and variety of lamps displayed made an enchanting spectacle. They may have been pleasant to look upon from a distance, but they had their drawbacks on closer acquaintance. One anonymous but frank Greek writer describes a feast at which "one could not enjoy the good things of the table until his indulgence in wine had made him indifferent to the stench of the lamps."

Castor oil, or a mixture of castor and linseed oils, were most commonly burned, but almost every kind of animal fat and vegetable oil has been tried by man, and whatever was most plentiful was most

commonly used. When the extraction of oil was rendered difficult through a lack of facilities, some extraordinary improvisations have been resorted to in order to provide a substitute for the more refined lamp. For example, in Northern Scotland, the carcass of a stormy petrel, an especially oily bird, has been used in place of a lamp. A wick was simply thrust into the bird's mouth. In Alaska the natives made a torch out of a fish held in a cleft stick.

The failure of the lamp to show much progress in development was not due to a want of experimentation. Efforts were continually being

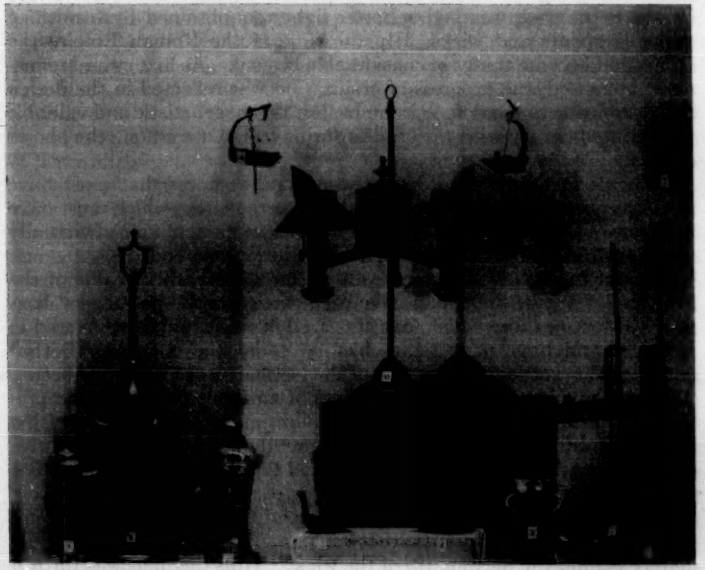

Photo by J. J. Barlon

Oil lamps of the 18th and 19th centuries.

made to make advances in lighting or to improve the convenience of the lamp. Sometimes it took the form of a huge metal or stone vase containing several pounds of liquid fat into which a wick of sorts was stuck. Such were the lamps used at large ceremonies. For domestic uses small lamps were made in great variety. When we examine the diversity of lamps which have been assembled from Greece, Egypt, Rome, and elsewhere, their similarity of principle is very striking. Whether made of clay, stone, or metal, the only distinguishing feature is the shape or the decoration.

The general idea is always the same. Beginning as a shallow bowl to hold the oil into which was laid a wick of pith or fiber to absorb the oil, centuries passed without any refinement of principle, which was exceedingly simple. The wick had to raise the oil by capillary attraction above the level of the supply in the reservoir so that it could receive the oxygen necessary to permit combustion.

Starting with the austere shape of a saucer (some were actually shells found on the seashore), they underwent slow and gradual change. The edges of the saucer were turned in, reducing the size of the opening but diminishing the risk of spilling the oil. Next, a spout was added in which the wick was laid. Better light was obtained by furnishing multiple spouts and wicks. By the time of the Roman Empire the lamps had become things of considerable beauty. As luxury and refinement increased, the progress of civilization was reflected in the design and decoration of the domestic lamps, but the most artistic and valuable were contemporary with the crudest forms still in use among the poorer classes.

When one thinks of all the pleasure and business that must have gone on after nightfall it is inexplicable that the lamp, which must have been one of the first things fashioned by ancient man, remained virtually unchanged until after the American Revolution had ended.

One of these inadequate lamps must have hung in the cabin of the *Mayflower;* a few others may have been carried in the passengers' baggage. Most of those which first appeared in this country originated in Holland, and have passed into history under the name of "Betty" lamps. The word is a corruption of the German *besser*, meaning better. Although much treasured by collectors of antiques they make no pretense to beauty. As a rule they are shallow metal scoops, often with a handle for lifting them, and not infrequently they are provided with chains so that when the lamp is suspended the chain can be adjusted to tilt the lamp in order to promote the flow of oil to the wick.

The Betty lamp had little to recommend it except its extreme simplicity. There was very little that could go wrong. The oil lay immediately behind the burning tip of the wick. Thus the light was thrown forward and sideways, leaving the room behind in shadow. Being a solid mass of soft cord or fibrous material, the wick drew the oil equally up its outer edge and interior, but as the oxygen necessary for combustion had access only to the outside, the wick burned imperfectly. The result was a smoky and unsteady flame. Further, unless the lamp was periodically tilted, the surface of the oil sank lower in the container, with a corresponding diminution of flame. The wick required frequent attention to remove the imperfectly consumed core. For the better performance of this latter service, some Betty lamps are provided with a small pick on a short chain.

Most Betty lamps were made of tin, but some are made of iron.

It is impossible to say which of those that have survived were made in this country and which were imported. The design is a common one, and metal workers had not yet begun to identify their individual handicraft with private marks.

The ancient type of lamp that lingered longest of all was the "crusie." This was the characteristic cabin lamp of the Pennsylvania pioneer. It comprised a pear-shaped open bowl with a flat band attached to the back, the upright finishing at the top in a swivel connecting it to a combined hook and spike, enabling the lamp to be hung from the roof or suspended against the wall. Some have two bowls, the upper one to contain oil and the lower to serve as a drip catcher. Occasionally the crusie has a lid and it is then indistinguishable from the Betty lamp.

Some cruisies found in Pennsylvania had "pushers" to keep the supply of semi-solid oil near the wick; others display attempts at ornament in the twist of the hook or upright shaft. A few crusies were still to be found in Bucks County, Pa., and in Tennessee as late as the middle of last century.

As long as the civilization of our country was based upon agriculture, all decorative details in the manufacture of household articles were of the simplest nature. In some of the older countries the decoration of lamps had already passed to the ornate stage, whereas American lamps were still plain and simple. The Hindus and the Moors had produced some exceedingly fine specimens of perforated brass in their temple and mosque lamps. These were designed to project intricate shadows as well as light, and to work upon the imagination of the worshipper.

At first the oil used in American lamps was extracted from the livers of small fish which were so abundant along the eastern shores. No effort was made to refine the oil. When the lamp was lighted it gave off a solid rank odor, which accounts for the expressed preference for candles. The Betty lamps were customarily placed or hung near the fireside with the optimistic hope that the disagreeable odor would find its way up the chimney, but so long as fish oil was used for lamps it is easy to understand why our ancestors preferred an outdoor life and went early to bed.

The feeble flicker of flame it provided and its accompanying odor must have furnished sufficient reason for promoting a search for substitutes. About the year 1750, either by accident, design, or impulse, someone tried the oil found in sperm and right whales, and found it was an illuminant vastly superior to any hitherto known. So marked was its superiority that this discovery led to a great expansion of the whaling industry. The demand for whale oil increased so rapidly that in a very short time the inshore supply of whales was exhausted. This promoted the creation of the great whaling fleet to extend the search for whales to furnish the increasing need. The fortunes of the Nantucket, Salem,

and New Bedford whalers were made while endeavoring to keep burning the oil lamps of a growing America.

Street lighting also benefitted from this discovery. Street lamps and, later, the important lights in lighthouses which must never fail for a moment, were all adapted to consume whale oil. But as the demand for the oil increased, the price began to climb. As the quest for oil progressed, the ravages upon the whales decreased their number and the whalers were forced to extend their voyages to more and more distant hunting grounds. These prolonged voyages, with the stronger ships required, only served to add to the sale price of the oil.

A few specimens of lard oil lamps have been preserved. These are much like glorified candles, as the heat of the burning wick reduces the lard to oil.

The general introduction of whale oil lamps did not drive the candle out of use. As old as the Pyramids, the candle retained its position in household lighting until the end of the eighteenth century. At Washington's anniversary party in 1817 the ballroom was lighted with 2000 candles. In that year a few candles could still be found lighting the streets of Philadelphia, although the oil lamp predominated. But man's knowledge of nature was rapidly expanding and, with his ability to overcome other obstacles, it would have been strange if his conquest of the darkness had been overlooked. With the nineteenth century, a new era in lighting had dawned.

In the year 1783, A Swiss chemist called Ami Argand introduced a great improvement in the oil lamp, which may be said to be the first scientific contribution to the art of lighting. All that Argand did was to give close attention to the combustion of the wick, and to provide devices that would ensure that the wick was more completely consumed. He realized that the disadvantage of the wick as it had been used for centuries resided in its restricted contact with the oxygen of the air. His first step, therefore, was to devise a wick that would expose a greater portion of its surface to the air. This he accomplished by bending a thin broad wick into a circle by enclosing it within two concentric tubes. Both sides of the circular wick were exposed to the air, providing for much better combustion and producing a brighter flame.

Argand went a step further by placing a glass chimney over the flame. This proved to be another improvement. Although glass had been known for centuries, no one had thought of using it to protect the wick of a lamp from draughts. When Argand tried it the effect was surprising. The chimney not only made the flame steadier, but it caused an updraft of air and, in consequence, brought a greater supply of oxygen to aid the combustion. It diminished the smoke.

The brightness of a flame determines the amount of light it furnishes. The Bunsen of your gas stove does not give a very bright light. The brilliancy of the flame is due to carbon particles heated to incandescence.

The animal and vegetable oils formerly used in lamps were rich in carbons but many carbon particles escaped as smoke before they could be heated to a stage when they became incandescent. Argand's chimney, with the updraft of air, so increased the temperature of the flame that many more carbon particles reached an incandescent state, all were made more luminous, and there was a better flame.

Argand's discovery merits emphasis. His devices brought to an end the open floating wick that had endured for centuries. From this time onward, all lamps were made with the oil in an enclosed container. By increasing the average household lamp to six-candle-power it gained great popularity, and all later lamps embodied Argand's principles. Even now, when city dwellers have become accustomed to improved forms of lighting, it should not be forgotten that millions of oil lamps are still being made and put to use where gas and electric mains have not yet been laid.

The marked increase in the knowledge of Nature and her ways that distinguishes the early years of the nineteenth century provided an impetus to the art of lighting. Once started it began to grow like a snowball rolling downhill. People had scarcely ceased to express their wonderment over Argand's improvements when another discovery was announced.

In the year 1802, the British people thought they had cause for rejoicing. Their rulers had just signed the Treaty of Amiens with the dreaded Napoleon. The occasion called for celebrations, decorations, and illuminations of an unusual order. Exceeding all other efforts was the spectacle presented by a group of factory buildings lighted by gas. This was regarded as a scene of extraordinary splendor. Judged by modern standards the illumination would provoke a smile of indulgence, but at that time it was unparalleled in its brilliance, and attracted a large concourse of curious people. Today the tendency in illumination, whether for ceremonial or private use, is to conceal the source of the light; we try to flood space with a light that is at once unobtrusive but all-pervading. But on this occasion, when gas-light was first presented to view, the multitude was not too critical. They were satisfied to see naked gas jets. After all, they were beholding a new marvel.

It is impossible to say when and by whom coal gas was first recognized as a suitable illuminant but William Murdock, an employee of the great James Watt, appears to have been the first to make it practical. He developed a method of baking coal to drive out the gas, and to collect it in retorts. His method was developed sometime about 1798, for that was the year when he first illuminated a portion of the firm's factory.

However, Murdock had not progressed to the stage of purifying his gas. Nor was his method of flattening the pipe to make a jet burner efficient. Householders might be willing to gape and gasp at the new wonder as an outdoor form of illumination, but they showed no ardor

about introducing the novelty of gas lighting into their homes. Many misconceptions existed about the new style of lighting. People could not easily grasp the idea of conveying gas through an iron pipe and then burning it at the outer end. They concluded that iron being a good conductor of heat, the pipe would become so hot that it would burn down the buildings in which it was installed. Other strange ideas developed, all about equal to one entertained by Sir Walter Scott who, in a letter to a friend, wrote: "There is a madman in London proposing to light London with—what do you think? Smoke!"

Nevertheless, the prejudices were slowly overcome and by the year 1815, when Napoleon had been overcome and the battle of New Orleans had been fought, gas-light was being so much talked about that a proposal was advanced to install a gas works in Philadelphia. The proposal provoked such vigorous protests against admitting poisonous gas that burned and gave off injurious vapors into dwelling houses, that it was promptly rejected. Baltimore, Boston, and New York were all ahead of Philadelphia in having gas light. Rembrant Peale lighted his art gallery in Baltimore by gas in 1816, and the effect was so gratifying that the city was induced to light a few of its streets by the same means in the following year. Boston was second in 1821, and New York followed in 1823.

Mr. Samuel Vaughan Merrick, founder of The Franklin Institute, returned from a visit to England where he had witnessed the great strides made in gas lighting, and through his earnest advocacy, a gas works was established in Philadelphia in 1835. At that time, the streets were generally lighted by whale oil lamps, but it was speedily shown that an equivalent gas light cost very little more and was infinitely more convenient. Torches, candles, and oil lamps had all been tried for street lighting, but none of these sources of light was regarded as satisfactory for the reason that each light was an independent unit. It was not until a central plant was established with its feeders connecting each lamp, that an entirely satisfactory street lighting system could be planned. The convenience could not be questioned. Moreover, whales were becoming scarcer, and the cost of whale oil was rising so rapidly that the question of cost was turning advantage in favor of gas.

However, there remained much prejudice against gas for house lighting. People were still suspicious of it, and afraid of its effect upon their health. The most radical improvement that had been introduced into lighting for a thousand years had to be satisfied with a slow progress. The gas had to be purified to make it acceptable and to provide better light and cleaner burners. Gas meters had to be devised so that consumers paid for no more gas than they actually burned.

The matter of improved burners became a grave concern to both producers and consumers. The first burner had been made by merely flattening the outlet of the pipe. This was improved by closing the

end of the pipe and piercing it with a series of small holes which made a flame in the shape of a coxcomb. Next the closed pipe was slit with a fine saw to make a flame shaped like a batswing. Finally, there came the fish-tail burner, which had two orifices so arranged that the jets of gas collided after emerging and spread out in the form of a fish's tail. Each of these improvements provided more light from a given volume of gas by exposing a larger light-emitting surface from a given flow. The law of radiation was not yet understood.

The first gas lamps all had open flames. Chimneys and globes were only gradually introduced to protect the flame from air currents.

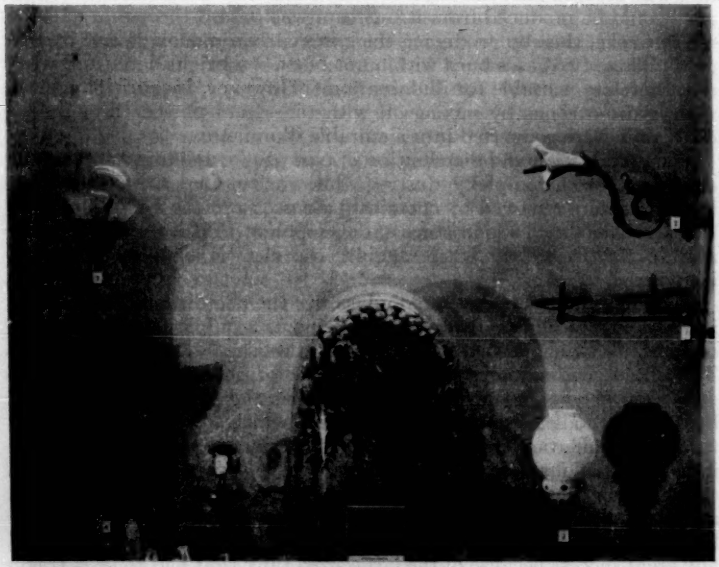

Photo by J. J. Barton

The gas-light period.

Generally speaking, all light sources which emit light solely because of their temperature (like candles, oil lamps, gas, and incandescent electric lamps) increase their brightness as the temperature is raised. The primary object in the search for an ideal gas burner should have been to increase the temperature of the source. But this principle was not understood at the time when gas was first introduced.

Because after it had been refined gas lighting offered definite advantages in cleanliness, convenience, and simplicity, its use became wide-

spread in those countries which had large supplies of coal. The fundamental principle of making the gas remained substantially the same as that originally devised by Murdock, although it underwent much improvement. This method is called destructive distillation. Briefly, the process consists of heating the coal in a retort until it fuses and its hydro-carbon vapors are freed. Some of the other materials in the coal, notably tar, were regarded as economic nuisances until late in the nineteenth century, when chemistry found that they were economic assets of high value.

After the hydro-carbon vapors had been extracted, the coke remained and was used as fuel, but soon it was found that the so-called "water gas" could be produced from it. Steam was passed through a mass of red-hot coke, thereby producing the gases carbon monoxide and hydrogen. These two gases burn with a hot but not a bright flame and were not, therefore, suitable for illumination. However, by enriching them with hydro-carbons by mixing oil with the steam passed through the coke, they were converted into a suitable illuminant.

The production and distribution of coal gas for lighting and heating developed into a complex industry. The various undesirable elements in the gas were removed by chemical processes and the art of illumination arrived at such a high stage of development that its engineers were able to fight a sturdy battle against the electric light when it was introduced.

Gas lighting became very popular once the chemists had succeeded in removing its sulphur content. Lighting as a public service qualified to meet all demands had been firmly established by the use of gas. The obvious advantages and convenience of gas piped through the streets to homes and workshops led to dissatisfaction with the other lighting methods available at the time and which were still required to furnish illumination in places not reached by the gas mains. This induced ingenious men to develop methods of providing gas lighting that was independent of any city supply. In the early days of railroads, many trains did not run after dark, and those which did run were unlighted. Coal gas was tried as an illuminant but did not meet with any great success because of the large sized tank that had to be carried. In the year 1880, Pintsch gas (called after the inventor) was introduced. This consisted of methane, hydro-carbons, and hydrogen, and could be compressed into tanks of much smaller dimensions. It became acceptable, not only on the railroads but also in remote places not easily accessible, such as light-buoys.

Other gases were used to less extent, and older readers will recall the extensive use made of acetylene gas, made from calcium carbide, in bicycle and early automobile lamps.

Meanwhile, the oil lamp had been given a great impulse. The growing scarcity of whales had driven the price of sperm oil to $1.75 cents a

gallon, higher than it had ever been before. Some substitute had to be found if the lamp was to be saved from becoming an expensive luxury. Colza oil had been used successfully in Europe, but as only a very small amount of rape seed (from which it was extracted) was grown in this country, not much progress could be looked for in that direction. The scarcity of other suitable oils drove experimenters to try mineral oil, which formerly had been valued only for its medicinal properties.

The early experiments to employ petroleum as an illuminating oil were not altogether successful. When Colonel E. L. Drake opened the first oilfield in Pennsylvania, the projected use of this type of oil was eagerly exploited, but the first experiences provoked alarm. The smell of the burning oil was well nigh unendurable and, worse than this, it proved to be highly unstable and even dangerous to use. Eventually, Colonel A. C. Ferriss produced a brand of kerosene that was odorless and not easily given to unpredictable explosion. When this was placed upon the market it received a warm welcome from those who were unable, for one reason or another, to use gas lighting. The tiny oil flames that had flickered through 8000 years and had given little more than one candlepower of illumination were replaced by kerosene lamps which produced ten candlepower or more.

The reception of the new illuminant was so cordial that within a few years the Patent Office had issued hundreds of patents on lamps especially designed to consume it.

The modest but useful candle was also improved by the discovery of petroleum's properties. Paraffin wax was extracted from the mineral oil, and was found to furnish a better basis for the candle wax. The original wax was given rigidity, its melting point was raised, and its brittleness diminished by the addition of stearine. The world was made a little brighter by the resultant candle. The candles of today vary somewhat in their composition, for much chemical knowledge has been expended upon them. If they burn better than their forerunners it is because great attention has been paid to minor details. For instance, wicks are now made to bend over so that their tips dip into the melted wax. This promotes complete combustion, abolishes the smoke, and does away with the need for trimming.

The discovery after so many centuries of such a satisfactory lamp would have assured for it universal adoption had not gas lighting superseded it. But the reign of the gas light was of short duration. It had scarcely assumed a dominating position when it was driven from the market by electricity. But gas did not surrender easily. A new invention gave it a new lease of life and an improvement in the gas light became a serious competitor for electric light, which was then struggling into the market.

About 1826 it had been learned that some substances glow more brilliantly when subjected to heat. This discovery had led to the devel-

opment of the Drummond or "lime-light," in which a button of lime was caused to glow with such vivid brilliance that it could be adopted for projecting its light over long distances, as in searchlights. It was by far the most powerful light source known. The success accorded the lime-light encouraged chemists to press their researches to disclose whether other substances possessed this property. It was found that some of the rare earths (such as zirconium, thoria, ceria, etc.) possessed this quality of glowing brilliantly when raised to a high temperature. Among those who investigated the properties of the rare earths was Auer von Welsbach. After ten years of research he introduced, in 1893, a commercial gas "mantle" which revolutionized gas lighting. This comprised a silk or cotton fabric impregnated with thoria and ceria. After burning away the fabric, the remaining ash gave off a brilliant glow in a hot gas flame.

It should be observed that in previous applications of gas lighting the flame of the burning gas provided the illumination. In this new application the illuminating property of the gas was of little account. It was merely the heating element to make the two rare earths furnish the illumination by their incandescence. A mantle added to a two candle power gas jet now increased the brilliance of the light to several times that candle power.

The first gas mantles were upright but, as improvements were made, it became possible to invert them, whereby the light was cast down where it was most needed. Many improvements were made in design to secure an accurate adjustment between the form of the flame and that of the mantle, so that the latter would be incandescent over its entire surface. The success of the incandescent gas mantle was instantaneous but its use was not of long duration. In spite of its efficiency as a light source it could not maintain its position against the new form of producing incandescence by passing an electric current through a conductor which offered a suitable resistance.

Even before Franklin embarked upon his electrical experiment investigators were familiar with the electric spark. It was never more than a transient glimpse of light until Volta invented his "pile," which furnished the first means of obtaining a continuous flow of current. Then, the great Humphrey Davy produced an electric arc. Using two carbon rods connected to an elaborate "pile" he found that, as the rods were drawn apart, an elongated spark of unsurpassed brilliance was produced. When the carbons were about three inches apart, the rise of the heated air caused the spark to bend in the form of an arch. Sparks produced in this manner continued to be called arcs long after vertical carbons came into use and the resultant sparks were no longer bent.

Probably the first use of the vivid light obtained by using a Voltaic battery outside the laboratory dates from 1845, when it was employed

at the Paris Opera House to create the effect of the rising sun. Its success in the role of Apollo encouraged other enterprising showmen to extend its field of usefulness by means of lenses and prisms to the production of luminous fountains, artificial rainbows, and lightning. The use of the electric spark for general lighting purposes was deferred until the year 1870, when the Gramme dynamo for furnishing adequate electric current was placed on the market. From that time, the electric arc attained prominence for the lighting of wide spaces.

The first commercial electric arcs were produced by passing a spark between two points of charcoal. Later, this material was replaced by carbon. Rods of carbon were made with a hollow center which was filled with a powder of soft carbon, to fulfill the dual purpose of enriching the flame and keeping the arc centered. The temperature at the points of the carbon rods was between 6000 and 7000 degrees Fahrenheit, by far the highest man-made temperature attained at that time. It was to provide the basis for the carbon-arc furnace.

Sweeping aside all the various experiments, and they were numerous, we find electric arc lamps being used to light the streets of Paris and London in 1876. The light source was a Jablochkov "candle" comprising two vertical carbon rods placed side by side but separated by an insulating material. It is said that these were first used for indoor lighting at John Wanamaker's store in Philadelphia on December 26th, 1876. They were almost immediately followed by Brush's more practical arc lamp. Neither of these was completely successful because they burned in the open air, and carbon consumes rapidly when burned in the air. These first arc lamps produced a blinding glare with much sputtering and hissing, but research brought improvement. L. B. Marks was one of the principal contributors to the development of the enclosed arc in 1893 when he showed that a small current of high voltage gave the best results when the carbons were enclosed in a glass globe. The combustion of the carbons was thereby diminished, so that the lights did not require the daily attention that had been devoted to them. A further result was a steadier light.

The day of the carbon arc soon passed and its place was taken by magnetite lamps and the mercury vapor arc. The former made use of a negative electrode of compressed magnetic iron oxide and titanium oxide. Although the powerful glare of the arc lamp was useful for lighting the streets of a city, it was not adapted to domestic use. Progress toward a household form of electric light had to await development along other lines. Much controversy centers around the original discovery of the modern incandescent electric lamp, since men were at work upon it in all civilized countries. The first patent covering an incandescent lamp using a carbon conductor in a vacuum was granted in England in 1845 to an American named J. W. Star. Like all the laboratory experiments which preceded it, this lamp was operated by a

high current at low voltage. Sawyer and Mann in England were using a platinum filament as early as 1877; Joseph Swan, another Englishman, carbonized a thread for use as a filament in a vacuum bulb in 1860. Edison introduced his first lamp, using a single loop of carbonized paper as filament, in 1879.

Edison is entitled, nevertheless, to the major share of credit for the incandescent lamp. He persevered along a trail that was studded with negative results, during which he tried over 6000 kinds of vegetable fibers before he found a filament that would offer sufficient resistance to the current to produce an adequate glow without being consumed. The principle of employing carbon was early recognized, but it was difficult to arrive at the proper combination of filament and environment. Edison concentrated upon the carbon filament because that substance was easily obtainable, and it had the desirable property of possessing a high melting point. But, as we have mentioned, carbon is readily consumed in the atmosphere, so that he was forced to develop the earlier method of heating his filament in a container from which the air was excluded. The familiar design of the electric lamp was the outcome of his efforts, a carbon filament glowing inside a vacuum bulb. The filament is no longer carbon, and the bulb is not evacuated, but it has followed the general design adopted by Edison.

Edison was the first to obtain a commercial lamp, but today we smile at the recollection of his filament. For the first nine years of its existence the Edison electric lamp had to rely upon a filament made of Japanese bamboo!

But Edison's share in the production of the modern electric light was not restricted to the development of the lamp. He had an advantage over his predecessors gained by the newly invented mercury air pump, which enabled him to obtain a high vacuum at a reduced cost. He was the first to use a fine filament which would become incandescent when a potential of 100 volts was applied. This combined with other elements of the Edison method (notably the three wire distribution and the feeder and main system) were as important as the improved lamp in making the electric light suitable for general use. By the time he had obtained the suitable lamp there was nothing of an electric system to go with it—dynamos, switches, switchboards, sockets, junction and service boxes, and special wiring had all to be developed. It is very largely to Edison's credit, and to the group of workers he had assembled, that all these necessary features were brought into being. To him, also, is due the system whereby the individual lamps could be controlled. Formerly, all the lights on a circuit had to be on or off. The "division" of the circuit to make individual lamp control possible was declared by reputable scientists at the time to be beyond the range of possibility.

After lighting his own workshop and home with the new light, Edison installed the first public system in Sunbury, Pa., in 1883. The original

bipolar direct current generator which was used in this three wire system is displayed in the museum of The Franklin Institute. This was driven by an Armington and Sims steam engine, and had a capacity of about 400 lamps of 16 candlepower.

For the next forty years there was a feverish competition to improve the electric lamp. Many changes for the design of bases and sockets were advanced before these were standardized. Better pumps had to be devised to arrive at the desired vacuum at which the carbon would not be consumed too quickly and so reduce the life of the lamp.

The first major steps in the improvement of the carbon filament had scarcely been taken when an entirely new material was introduced as a filament. The German von Bolton had found that the metal tantalum could be drawn into a fine wire with a high melting point and other properties which made it an attractive substitute for carbon. As the resistance of the tantalum to electric current was low, the filament had to be made correspondingly long, and this led to the introduction of glass supports inside the lamp on which a very long filament was draped. This lamp proved to possess a higher efficiency but it had defects which, doubtless, would have been removed by further research, had not experiments shown that tungsten was even better than tantalum.

Tungsten was found to have unrivalled properties for a lamp filament. Lamps using this metal for filament were introduced in 1907 by two Austrian chemists, Just and Hanaman. Unfortunately, the method of manufacture produced a very fragile lamp that was easily damaged by shock, and which had a very short life. Finally, drawn tungsten wire was rendered possible by the research of William D. Coolidge in Schenectady in 1911. When the handicap of extreme fragility had been overcome it was learned that the high temperatures to which the tungsten was subjected caused it to evaporate and to blacken the bulb. It was apparent that if the unmistakable advantages of tungsten were to be enjoyed the vacuum bulb would have to be abandoned in favor of one containing an inert gas that would check the chemical action of the hot filament. A great amount of painstaking research was necessary before the exact length and thickness of filament was determined, and the correct combination of gases was ascertained. This work was largely accomplished by Irving Langmuir, also of Schenectady, who doubled the efficiency of the incandescent lamp with his gas-filled spiral filament lamp, produced in 1913.

The electric lamp filament of today is a very fine tungsten wire coiled in the form of a helix to obtain the required resistance within a small area. These coils are works of art, calling for the utmost accuracy in construction. The filament of a 60 watt lamp, for example, consists of a wire 0.0019 inch in diameter and about twenty inches long. This is coiled in about 1200 turns, reducing its length to rather less than $3\frac{1}{2}$ inches. This is converted into a coil about $\frac{1}{2}$ inch long, thus providing

in a very small area a high concentration of material. Two gases are introduced into the bulb, nitrogen and argon, so that it is no longer a vacuum. The inside of the bulb is frosted to diffuse the light without appreciable loss.

We have followed the development of the lamp filament without digression, although there should have been several to make our story complete, but it should be pointed out here that the progress of light production by electricity followed a straight line—to increase the temperature of the light source. Certain practical considerations place limitations upon the filament temperature. Obviously, the temperature at which the filament can be operated is limited by its melting point but, to obtain a lamp that will maintain a uniform light over a long period of time, a great deal more than this had to be considered. Lamps for household use are now designed to burn for 1000 hours at a specified candlepower, provided the correct voltage is employed, but a great diversity of lamps had to be made and rejected before the present stage of efficiency was reached.

Some of the stages in the advance may appear to be slight but, even so, they represent colossal savings to the consumers. The number of electric lamps in use in the United States is so great that a new lamp with an increased efficiency of 10 per cent will save more than ten million dollars in the nation's light bill.

No one ventures to predict that the maximum efficiency in the metal filament lamp has been reached, but the tendency today is to follow the path that leads in the direction of vapor, gaseous discharge, and fluorescent lamps. Some of the more recent developments are making use of a principle that has not been mentioned here—an electrical discharge to cause gases to glow brightly. We have seen that some metals when raised to high temperature give off a bright light. Certain gases composed of fine particles floating in suspension possess the same property of illumination when heated. Three chemical elements, mercury, sodium, and neon, are most conspicuous in the production of light by gaseous conduction. Although sodium is a solid and mercury is fluid, both are vaporized before use in illumination.

The first practical mercury vapor lamp, made by Peter Cooper-Hewitt in 1902 was a milestone in the story of lighting, marking a departure from either flame or filament as a light source. This type of lamp has undergone much improvement and has become the competitor of the incandescent filament lamp in some applications. It is extensively used in industrial lighting. The absence of red from the light it emits limits its uses, because some colors disappear and others are distorted under its light.

The sodium vapor lamp has, theoretically, double the efficiency of the tungsten filament, but its light source is relatively so large that it cannot be controlled as effectively as the more concentrated tungsten

filament. However, these lamps find uses on highways and where the size of the lamp does not limit its use.

The neon lamp is familiar in advertising signs because of its distinctive orange-red color. It does not figure in our story of domestic lighting except to illustrate how the knowledge of light sources has expanded. Moreover, further research into gaseous discharge lamps led to a great increase of knowledge relating to pressures within the bulb. It was knowledge derived from the low pressure mercury arc lights that made possible an application which marks a departure from every other concept of lighting. This was the development of the fluorescent lamp.

All previous light sources had involved the element of temperature. Particles of matter had to be energized by heat until their activity caused them to glow and to emit light. To make more light without wasting energy in heat was an illusive vision pursued by the lighting engineer. He always had the tantalizing spectacle of seeing this miracle performed in nature whenever he watched the fireflies weaving their invisible patterns against the dark background of a night in June. Numerous other little insects and fishes possessed this ability to produce cold light, and their singular ability was a perpetual challenge to the lighting engineer. He began to search for an artificial form of this cold light.

When we project the spectrum of a light source on a white surface the colors of the rainbow are spread before our eyes. Beyond the reds at one end and the violet at the other end we can see nothing. But if we place some substances just beyond the rainbow at the violet end, we perceive that they are invested with the magic property of glowing with unexpected brilliance. The explanation of this phenomenon is to be found in the presence of ultra-violet radiation, not discernible by human vision.

These substances which possess the property of glowing (or fluorescing) are frequency changers in the realm of radiant energy. They absorb the ultra-violet energy of certain wavelengths and emit longer wavelengths of lower frequency. When the latter are within the range of visibility they stimulate our sensation of color or brightness. This property of ultra-violet radiation to make certain substances fluoresce had already been used to detect forgeries in paintings and documents before it was successfully applied to lighting. The decisive feature of its application was the discovery that the short wavelengths of ultra-violet radiation to which the substances were sensitive could be produced abundantly by the low pressure mercury arc. Once this was apparent progress became rapid.

The insides of glass tubes are painted with powders of various fluorescent materials, each powder producing its individual color. The powders may be mixed like paints to blend for the production of required tones, or to furnish a close resemblance to sunlight. Everyone is satis-

fied with that clear "whiteness" that streams through the windows at midday, but it was unfortunate that everyone had become unconsciously accustomed to associate a yellow tinge with all artificial light. Oddly enough, it was this accommodation to artificial yellow light that proved the principal stumbling block to the adoption of the "daylight" lamp rendered possible by fluorescent light. This prejudice had to be overcome before the better lighting, approximating to natural daylight, was welcomed.

The coolness of the fluorescent light is partly psychological, but it is none the less real. For ordinary reading or clerical work a light of 100 foot candlepower is necessary. But the heat from this when the light source is a tungsten filament incandescent lamp can become very uncomfortable. The heat is not appreciable with a fluorescent lamp of equal light power.

Thus we have today, lamps that are relatively small (tungsten filament) but of high brightness, and others that are relatively large (fluorescent) and of low brightness. Between these two types of lamp we are better able than men have ever been to select an individual lamp for each and every purpose. At a lighting exhibition 2000 lamps of various sizes and uses may be displayed, illustrating the wide range from which a choice can be made.

The continual progress made in brightening the lamp of civilization is a reflection of the progress made through science. Our modern light is only a century old. If we look behind us into the past we see the lights growing dimmer and dimmer as they recede in time, but if we look sympathetically we will perceive that these dim lights all have a meaning. Some of those old lights are not so far behind us, especially the old, smoking, flaming torch. If we turn our backs on this vista of the past and peer into the future we shall see another torch lighting the way—the Torch of Knowledge, handed from scientist to scientist as generations succeed each other.

A METHOD FOR SOLVING THIRD AND HIGHER ORDER NONLINEAR DIFFERENTIAL EQUATIONS

BY

Y. H. KU [1]

INTRODUCTION

In a previous paper (1)[2] an acceleration plane method has been given for the solution of a second-order nonlinear differential equation of the form

$$\ddot{x} = f(\dot{x}, x) + f_1(x) = F(t) \tag{1}$$

where $\ddot{x} = d^2x/dt^2$, $\dot{x} = dx/dt$, x = displacement, and t = time. Extension of the method (2) has been made to the solution of a nonlinear differential equation of the form

$$\phi(\dot{x})\ddot{x} + f(\dot{x}, x) + f_1(x) = F(t). \tag{2}$$

Two examples are given for $F(t) = \sin t$ and $F(t)$ equal to a rectified sine wave. It is also mentioned that the method can apply to the more general form involving time-varying parameters,

$$\phi(\dot{x}, t)\ddot{x} + f(\dot{x}, x, t) + f_1(x, t) = F(t). \tag{3}$$

In the present paper it is first shown that for $\dddot{x} = d^3x/dt^3$,

$$\dddot{x} + \phi(\dot{x})\ddot{x} + f(\dot{x}, x) + f_1(x) = F(t) \tag{4}$$

can be solved by a generalized phase-plane method. The method is then extended to the general case of a higher order nonlinear differential equation,

$$x^{(n)} + \phi_{n-1}x^{(n-1)} + \cdots + \phi_2\ddot{x} + f(\dot{x}, x) + f_1(x) = F(t) \tag{5}$$

where $x^{(n)} = d^nx/dt^n = n$th derivative, $x^{(n-1)} = (n-1)$th derivative, $\cdots$ $x^{(2)} = \ddot{x} = d^2x/dt^2$ = second derivative, $x^{(1)} = \dot{x} = dx/dt$ = first derivative, x = displacement, t = time, ϕ_{n-1} is a nonlinear function involving $x^{(n-1)}$ and lower derivatives, ϕ_2 is a function of $\ddot{x}$, $\dot{x}$, and x, and $f(\dot{x}, x)$ and $f_1(x)$ are given functions of $\dot{x}$ and x.

According to the discussion in reference 2, all the functions ϕ, f, and f_1 can include time functions without modifying the general method of solution. The same thing can be said in extending the solution of

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[2] The boldface numbers in parentheses refer to the references appended to this paper.

Eq. 5 to

$$x^{(n)} + \phi_{n-1}x^{(n-1)} + \cdots + \phi_2 x^{(2)} + \phi_1(\dot{x}, x, t) + \phi_0(x, t) = F(t) \quad (6)$$

where

$$\phi_{n-1} = \phi(x^{(n-1)}, x^{(n-2)}, \cdots, t).$$

SOLUTION OF A THIRD ORDER NONLINEAR EQUATION

We shall start from Eq. 4 and let the third-order time derivative d^3x/dt^3 at any given instant be denoted by h, where h is in general a function of $\ddot{x} = g$, $\dot{x} = v$, x and t, g standing for acceleration, v for velocity, and x representing displacement. Thus from Eqs. 3, 4 and 6,

$$\dddot{x} = h(\ddot{x}, \dot{x}, x, t) = F(t) - \phi(v, t)g - f(v, x, t) - f_1(x, t). \quad (7)$$

This h-function may be termed the time-rate of acceleration or dg/dt.

Since $\dddot{x} = d\ddot{x}/dt = dg/dt$ and $\ddot{x} = d\dot{x}/dt = dv/dt$, dividing all members of Eq. 7 by $\ddot{x} = g = dv/dt$ gives

$$\frac{dg}{dv} = \frac{h(g, v, x, t)}{g} = \frac{F(t) - \phi(v, t)g - f(v, x, t) - f_1(x, t)}{g}. \quad (8)$$

This represents a generalized phase-plane equation determining the slope of the $g - v$ trajectory everywhere according to the ratio h/g, just as in solving a second-order nonlinear differential equation the phase-plane equation $dv/dx = g/v$ determines the slope of the $v - x$ trajectory, as shown in reference 1.

Physically, Eq. 8 can be interpreted as follows: The rate of change of acceleration with respect to velocity at any instant is determined by the ratio of time-rate of acceleration h to acceleration g at the same instant. In evaluating Eq. 8, we can follow the physical boundary conditions. In fact, we are tracing the trajectories of h, g, v from our initial condition (say, $x = x_0$ at $t = 0$) through a cycle or several cycles of oscillations until we reach the final steady-state condition, if the system is stable.

In reference 1, we have plotted the g function (acceleration) against x (displacement) for constant values of v (velocity). These curves may be termed the equi-velocity curves on the $g - x$ plane (or the acceleration plane). The h function (rate of acceleration) may be plotted against v for pairs of values of g and x, or it may be plotted against v and x in a three-dimensional plot for constant values of acceleration g. However, the obvious difficulty of a three-dimensional plot should not prevent us from getting simplified procedures directly from Eq. 8. Such a procedure will be outlined in the next section.

Assuming that there is a way of constructing the $g - v$ trajectory such that its slope is given by h/g according to Eq. 8, we can tabulate

the pairs of values of g and v read off the $g - v$ trajectory. Knowing these pairs of values, we can get the ratio g/v and construct the $v - x$ trajectory according to the relation

$$\frac{dv}{dx} = \frac{g(h, v, x, t)}{v} = \frac{g}{v}. \tag{9}$$

The $v - x$ trajectory is thus plotted on the velocity phase-plane (or the $v - x$ plane), while the corresponding $g - x$ trajectory can be plotted on the acceleration phase-plane (or the $g - x$ plane). With the $v - x$ trajectory constructed, it is a routine matter to construct the $x - t$ curve and the $v - t$ curve. It may be also of interest to construct the $g - t$ curve and the $h - t$ curve to show the instantaneous values of acceleration and its rate of change.

CONSTRUCTION OF ACCELERATION-VELOCITY TRAJECTORY

In order to illustrate the method of construction of an acceleration-velocity trajectory (or a $g - v$ plot) from Eq. 8, we shall take a simple example, keeping in mind that in making the construction we do not impose any restriction on the h-function defined by Eq. 7. Let the third-order differential equation be given by

$$\dddot{x} + a_2\ddot{x} + a_1\dot{x} + a_0x = F = \text{const.} \tag{10}$$

According to Eq. 8, there is obtained,

$$\frac{dg}{dv} = \frac{F - a_2g - a_1v - a_0x}{g} = \frac{h(g, v, x)}{g}. \tag{11}$$

For $a_2 = 2$, $a_1 = 2.25$, $a_0 = 1.25$ and $F = 1.25$, Eq. 11 becomes

$$\frac{dg}{dv} = \frac{1.25 - 2g - 2.25v - 1.25x}{g} = \frac{h}{g}, \tag{12}$$

where

$$h(g, v, x) = 1.25 - 2g - 2.25v - 1.25x. \tag{12a}$$

Similar to the construction of the $v - x$ trajectory explained in reference 1, we shall assume an initial slope for the $g - v$ trajectory to be constructed according to Eq. 12. From physical considerations, the initial conditions are given as follows: at $t = 0$, $x_0 = 0$, $v_0 = 0$, $g_0 = 0$, and $h_0 = 1.25$ according to Eq. 12a. The initial slope, as given by $h_0/g_0 = 1.25/0 = \infty$, is infinite, and hence the $g - v$ trajectory starts vertically up from the origin. We shall start with an almost vertical slope, say, a slope of $+40$. Taking a small interval $\Delta v = 0.0015$ along the horizontal axis to the right of the origin, the change in acceleration Δg is then, according to the assumed slope ($dg/dv = 40$), equal to

$40(\Delta v) = 0.06$. Notice that we can as well start from $\Delta g = 0.06$ and then obtain $\Delta v = \Delta g/40 = 0.0015$. Since $v_0 = 0$, the velocity at the end of the chosen interval will be $v_1 = v_0 + \Delta v = 0.0015$, and the average value of velocity during this small interval can be taken as $v_{av} = v' = \frac{v_0 + v_1}{2} = 0.00075$. Similarly, since $g_0 = 0$, the acceleration at the end of the same interval is given by $g_1 = g_0 + \Delta g = 0.06$, and the average value of acceleration during this interval is $g_{av} = g' = \frac{g_0 + g_1}{2} = 0.03$. As a first estimate, the average value of rate of acceleration h_{av} or h' can be calculated from

$$\frac{\Delta g}{\Delta v} = \frac{h_{av}}{g_{av}} = \frac{h'}{g'}.$$

That is, $h' = (40)g' = 40(0.03) = 1.20$.

In order to find the value of h' from Eq. 12*a*, it is necessary to find the corresponding value of x_{av} or x'. Since, according to Eq. 9, the slope of the $v - x$ trajectory is given by the ratio g/v, we have

$$\frac{\Delta v}{\Delta x} = \frac{g_{av}}{v_{av}} = \frac{g'}{v'}.$$

Since $g'/v' = 0.03/0.00075 = 40$, $\Delta x = \Delta v/40 = 0.0015/40 = 0.000037$. The value of displacement x at the end of the interval will be $x_1 = x_0 + \Delta x = 0.000037$, and the average value during the interval will be

$$x_{av} = x' = \frac{x_0 + x_1}{2} = 0.000018.$$

Substituting the values of g', v', and x' into Eq. 12*a*,

$$h' = 1.25 - 2g' - 2.25v' - 1.25x' = 1.1883.$$

The correct slope at the middle point of the interval is given by

$$\frac{dg}{dv} = \frac{h'}{g'} = \frac{1.1883}{0.03} = 39.61$$

which checks reasonably well with the assumed slope.

From the calculation of h' according to Eq. 12*a*, it is seen that in this numerical example, the contribution of $1.25x$ is less than 0.002 of a per cent and does not affect the final result $h' = 1.1883$ with four decimal points. So with a chosen value of $\Delta g = 0.06$ or $g' = 0.03$, the slope can be easily adjusted to get a proper value of Δv so that $2.25v'$ will modify the value of h' in such a way that the ratio h'/g' agrees with the assumed slope, $\Delta g/\Delta v$.

Notice that the proper slope for a chosen small interval is to be taken as the true slope for a point midway between the two end points of the interval. Thus for the $g - v$ trajectory under construction, the slope dg/dv is to be constructed at g' and v' according to the ratio h'/g'. While g' represents the acceleration value at the mid-point from g_0 to g_1 on the assumed slope line, v' represents the velocity value at the mid-point from v_0 to v_1 on the same assumed slope line. When the

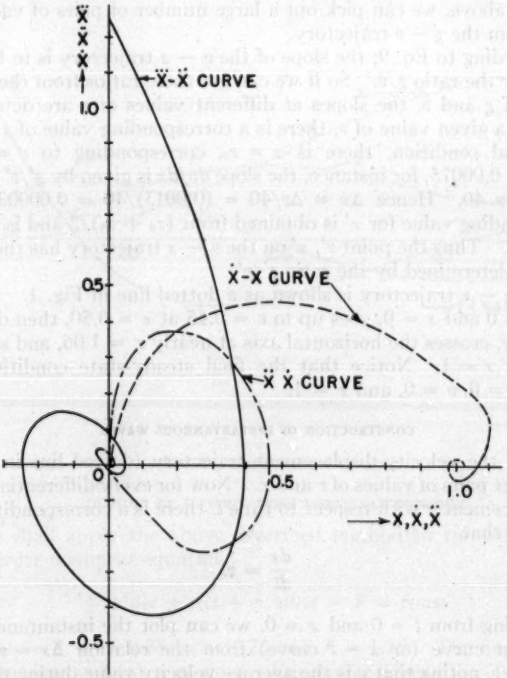

FIG. 1. Generalized phase-plane plots for third-order equation.

assumed slope agrees with the ratio h'/g', the g', v' pair falls on the proper $g - v$ trajectory.

The $g - v$ trajectory is shown in the dash and dotted line curve in Fig. 1. The $h - g$ trajectory is also shown in Fig. 1 as a solid line. It is seen that the rate of acceleration h starts from 1.25 at $g = 0$, crosses the horizontal axis at nearly $g = 0.4$, swings back from the

fourth quadrant to the third quadrant, then goes up to the second quadrant, and spirals toward the origin. The $g - v$ trajectory starts from $g = 0$ and $v = 0$, crosses the horizontal axis at nearly $v = 0.46$, and spirals toward the origin as shown.

CONSTRUCTION OF VELOCITY-DISPLACEMENT TRAJECTORY

Since the acceleration-velocity trajectory for a given physical (or mathematical) problem has been constructed according to the procedure outlined above, we can pick out a large number of pairs of values of g and v from the $g - v$ trajectory.

According to Eq. 9, the slope of the $v - x$ trajectory is to be determined by the ratio g/v. So if we can get these ratios from the pairs of values of g and v, the slopes at different values of v are determined. Now for a given value of v, there is a corresponding value of x. From the initial condition, there is $x = x_0$, corresponding to $v = v_0$. At $v = v' = 0.00075$, for instance, the slope dv/dx is given by $g'/v' = 0.03/0.00075 = 40$. Hence $\Delta x = \Delta v/40 = (0.0015)/40 = 0.000037$. The corresponding value for x' is obtained from $(x_0 + x_1)/2$ and is equal to 0.000018. Thus the point v', x' on the $v - x$ trajectory has the correct slope as determined by the ratio g'/v'.

The $v - x$ trajectory is shown as a dotted line in Fig. 1. It starts from $v = 0$ and $x = 0$, goes up to $v = 0.45$ at $x = 0.50$, then decreases gradually, crosses the horizontal axis at nearly $x = 1.06$, and spirals to $v = 0$ at $x = 1$. Notice that the final steady-state conditions are: $h = 0$, $g = 0$, $v = 0$, and $x = 1$.

CONSTRUCTION OF INSTANTANEOUS WAVES

From the velocity-displacement trajectory (dotted line in Fig. 1), we can get pairs of values of v and x. Now for every differential change of displacement x with respect to time t, there is a corresponding value of v such that

$$\frac{dx}{dt} = v. \tag{13}$$

Starting from $t = 0$ and $x = 0$, we can plot the instantaneous displacement curve (or $x - t$ curve) from the relation $\Delta x = v(\Delta t)$, or $\Delta t = \Delta x/v$, noting that v is the average velocity value during the small interval. The proper value of v can be read off the $v - x$ trajectory for a chosen Δx from an initial value of x, say, $x = x_0$.

Since for every value of x there is a corresponding value of v on the $v - x$ trajectory, it is straight-forward to plot the instantaneous velocity $(v - t)$ curve.

Similarly, from the $v - t$ curve and the $g - v$ trajectory, the instantaneous acceleration $(g - t)$ curve can be obtained. Finally, from the $g - t$ curve and the $h - g$ trajectory, the instantaneous acceleration-

rate ($h - t$) curve can be plotted. Thus we have

$$\frac{dv}{dt} = g; \quad \frac{dg}{dt} = h. \tag{14}$$

These instantaneous waves are shown in Fig. 2.

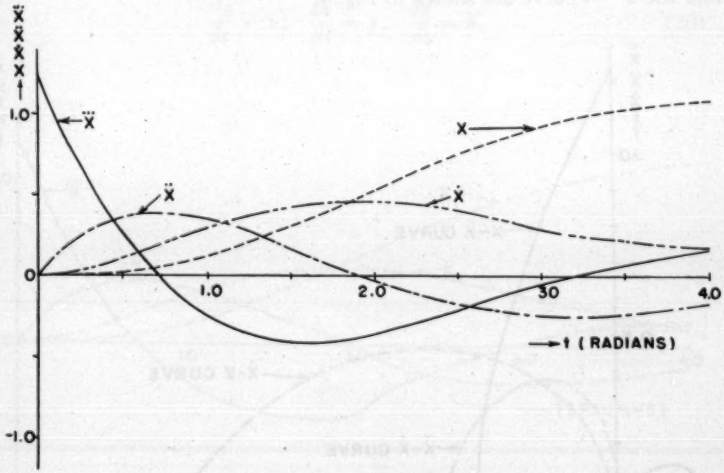

FIG. 2. Instantaneous waves.

SOLUTION OF A THIRD-ORDER NONLINEAR EQUATION

We shall apply the above described method to the solution of a third-order nonlinear equation

$$\dddot{x} + a_2\ddot{x} + a_1\dot{x} + a_0 \sin x = F = \text{const.} \tag{15}$$

where $a_2 = 2$, $a_1 = 2.25$, $a_0 = 1.25$ and $F = 1.25$. The nonlinear element appears as $f_1(x) = a_0 \sin x$. Equation 8 gives

$$\frac{dg}{dv} = \frac{F - a_2 g - a_1 v - a_0(\sin x)}{g} = \frac{h}{g}. \tag{16}$$

If the forcing function F is suddenly applied at $t = 0$, the initial rate of acceleration is given by $h_0 = F = 1.25$, for $g_0 = 0$, $v_0 = 0$, and $x_0 = 0$. The initial slope of the $g - v$ trajectory is about the same as that in the preceding case, as $\sin x$ is almost equal to x for small values of x. As x

gets larger, sin x is different from x, and the slope dg/dv will be different from that in the preceding case, for the same value of x. Finally, in the steady-state, sin $x = 1$, or $x = \pi/2 = 1.57$ is the focal point, while in the preceding case $x = 1$ is the focal point.

The $h - g$ trajectory, the $g - v$ trajectory, and the $v - x$ trajectory are shown in Fig. 3. The $h - t$ curve, the $g - t$ curve, the $v - t$ curve, and the $x - t$ curve are shown in Fig. 4.

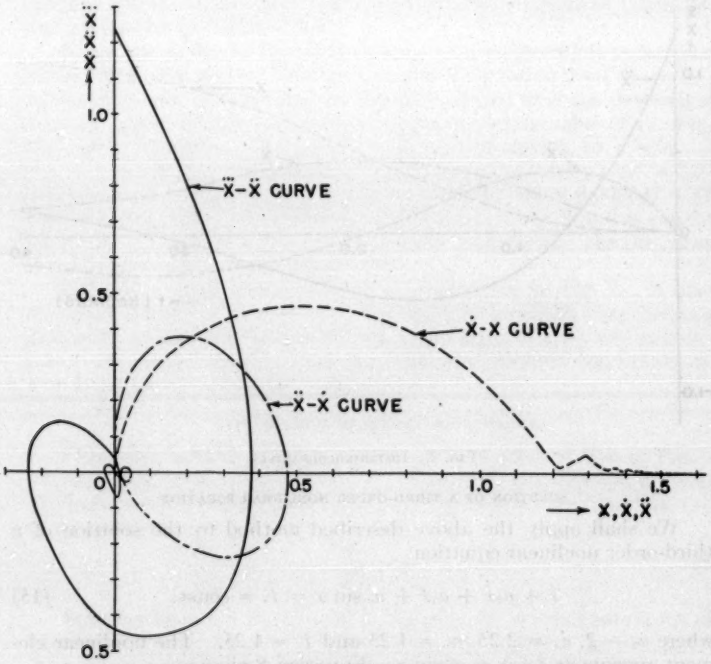

FIG. 3. Generalized phase-plane plots for third-order nonlinear equation.

It should be pointed out that the method applies to any nonlinear differential equation of the general form

$$\dddot{x} + a_2(\dot{x})\ddot{x} + a_1(x)\dot{x} + a_0 f_1(x) = F(t) \tag{17}$$

or

$$\dddot{x} + \phi(\ddot{x}, \dot{x}, x, t) + f(\dot{x}, x, t) + f_1(x, t) = F(t). \tag{18}$$

The phase-plane equations are

$$\frac{dg}{dv} = \frac{h}{g}; \quad \frac{dv}{dx} = \frac{g}{v}. \tag{19}$$

The instantaneous wave equations are

$$\frac{dx}{dt} = v; \quad \frac{dv}{dt} = g; \quad \frac{dg}{dt} = h. \tag{20}$$

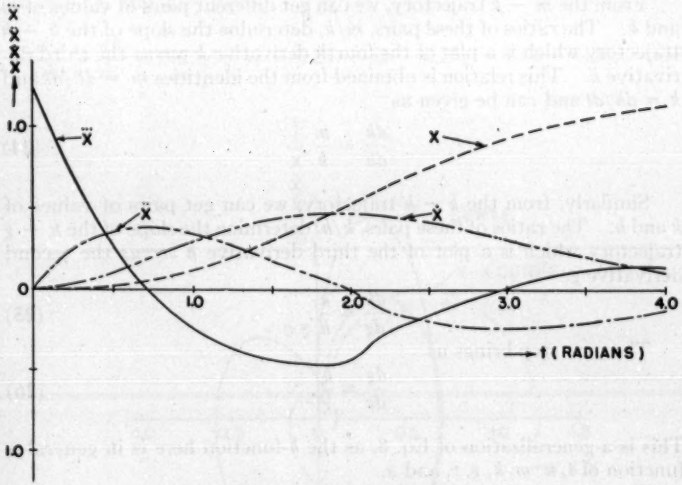

FIG. 4. Instantaneous waves.

EXTENSION TO HIGHER ORDER NONLINEAR EQUATIONS

Take a sixth-order nonlinear differential equation, such as

$$\frac{d^6x}{dt^6} + \phi_5\frac{d^5x}{dt^5} + \phi_4\frac{d^4x}{dt^4} + \phi_3\frac{d^3x}{dt^3} + \phi_2\frac{d^2x}{dt^2} + f\left(\frac{dx}{dt}, x\right) + f_1(x) = F(t). \tag{21}$$

Let $d^6x/dt^6 = n$, $d^5x/dt^5 = m$, $d^4x/dt^4 = k$, $d^3x/dt^3 = h$, $d^2x/dt^2 = g$, and $dx/dt = v$. Then

$$\frac{d^6x}{dt^6} = \frac{dm}{dt} = n(t, m, k, h, g, v, x) = F(t) - \phi_5 m - \phi_4 k - \phi_3 h - \phi_2 g - f(v, x) - f_1(x). \tag{22}$$

Dividing all members of Eq. 22 by $d^5x/dt^5 = dk/dt = m$,

$$\frac{dm}{dk} = \frac{n(t, m, k, h, g, v, x)}{m}, \tag{23}$$

where $n(t, m, k, h, g, v, x)$ is as defined by Eq. 22. This higher-order phase-plane equation (Eq. 23) determines the $m - k$ trajectory which is a plot of the fifth derivative m *versus* the fourth derivative k on the $m - k$ phase-plane.

From the $m - k$ trajectory, we can get different pairs of values of m and k. The ratios of these pairs, m/k, determine the slope of the $k - h$ trajectory which is a plot of the fourth derivative k *versus* the third derivative h. This relation is obtained from the identities $m = dk/dt$ and $k = dh/dt$ and can be given as

$$\frac{dk}{dh} = \frac{m}{k}. \tag{24}$$

Similarly, from the $k - h$ trajectory, we can get pairs of values of k and h. The ratios of these pairs, k/h, determine the slope of the $h - g$ trajectory which is a plot of the third derivative h *versus* the second derivative g.

$$\frac{dh}{dg} = \frac{k}{h}. \tag{25}$$

The next step brings us

$$\frac{dg}{dv} = \frac{h}{g}. \tag{26}$$

This is a generalization of Eq. 8, as the h-function here is in general a function of t, n, m, k, g, v, and x.

The slope of the $v - x$ trajectory is then given by

$$\frac{dv}{dx} = \frac{g}{v} \tag{27}$$

which is a generalization of Eq. 9, as the g-function depends on t, n, m, k, g, v, and x.

The instantaneous displacement $(x - t)$ curve is then obtainable from

$$\frac{dx}{dt} = v(t, n, m, k, h, g, x). \tag{28}$$

From the $x - t$ curve and the $v - x$ trajectory, we can get the $v - t$ curve, as explained before. Similarly, from the $v - t$ curve and the $g - v$ trajectory, we can get the $g - t$ curve. Thus the $g - t$, $h - t$, $k - t$, $m - t$, and $n - t$ curves can be obtained from Eqs. 22 to 28.

EXAMPLE OF A FOURTH-ORDER DIFFERENTIAL EQUATION

Let us take a simple example for a fourth-order differential equation, keeping in mind that nonlinear elements can replace any of the constant coefficients without any change in the method of solution as outlined previously.

$$\dddot{\dddot{x}} + a_3\dddot{x} + a_2\ddot{x} + a_1\dot{x} + a_0 x = F. \tag{29}$$

Using the previous notations, and letting $a_3 = 3$, $a_2 = 8.25$, $a_1 = 7.5$, $a_0 = 6.25$, and $F = 6.25$,

$$\frac{d^4x}{dt^4} = \frac{dh}{dt} = k(h, g, v, x) = 6.25 - 3h - 8.25g - 7.5v - 6.25x. \tag{30}$$

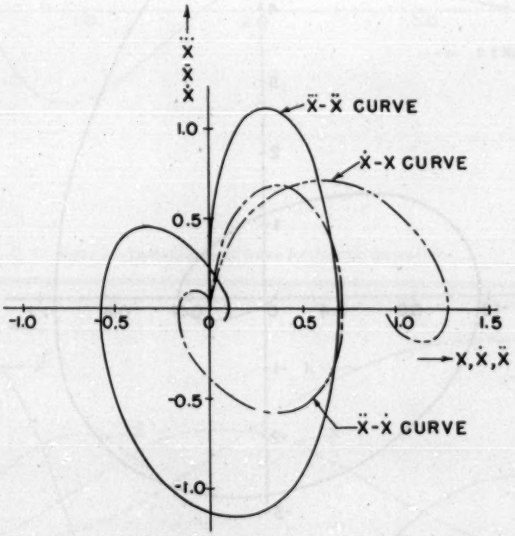

FIG. 5. Generalized phase-plane plots for fourth-order equation.

Dividing all members of Eq. 30 by $d^3x/dt^3 = dg/dt = h$,

$$\frac{dh}{dg} = \frac{k}{h}. \tag{31}$$

The $h - g$ trajectory constructed according to Eq. 31 is shown as a solid line in Fig. 5. Since for every value of h there is a corresponding value

of k, according to Eq. 31, the $k - h$ trajectory can also be constructed as shown in Fig. 6. From the $h - g$ trajectory, we can get pairs of values of h and g. Then, from

$$\frac{dg}{dv} = \frac{h}{g} \tag{32}$$

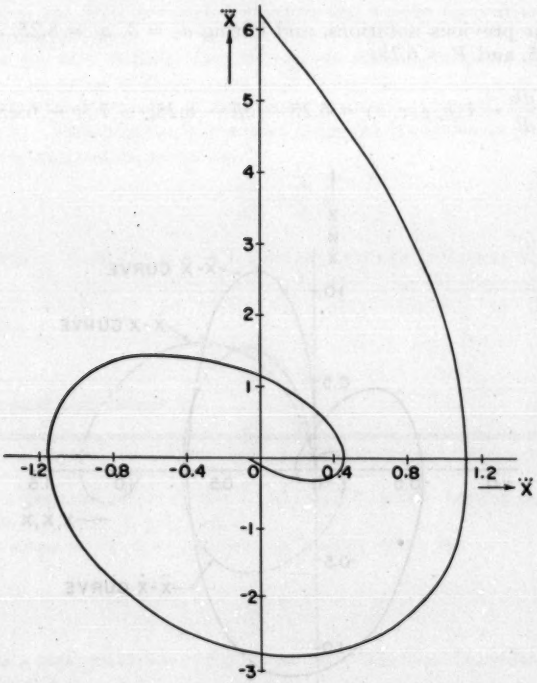

FIG. 6. Plot of fourth derivative *versus* third derivative.

we can construct the $g - v$ trajectory, as shown (dash and dotted line) in Fig. 5. Similarly, the $v - x$ trajectory can be obtained from the $g - v$ trajectory by the relation

$$\frac{dv}{dx} = \frac{g}{v}. \tag{33}$$

The $v - x$ trajectory is also shown in Fig. 5.

The $k - t$ curve is shown in Fig. 7. The $h - t$, $g - t$, $v - t$, and $x - t$ curves are all shown in Fig. 8.

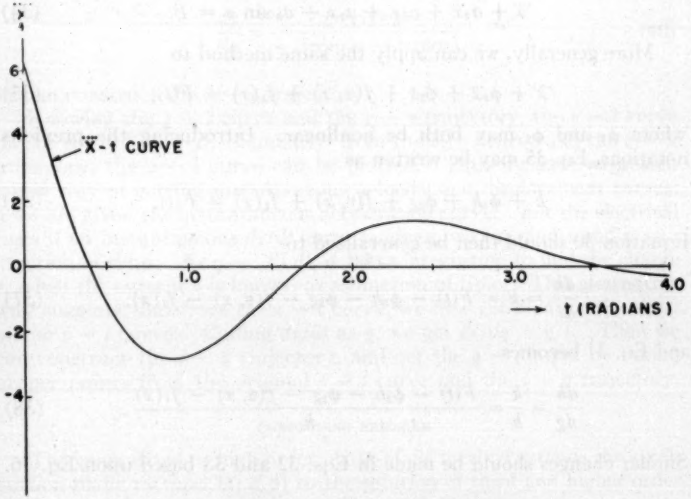

FIG. 7. Instantaneous wave for fourth derivative.

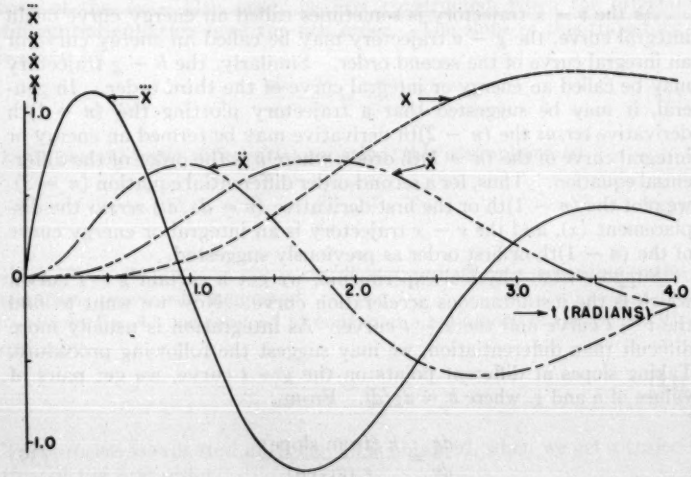

FIG. 8. Instantaneous waves.

A similar set of plots can be made with nonlinear elements introduced into Eq. 29, say,

$$\dddot{x} + a_3\ddot{x} + a_2\ddot{x} + a_1\dot{x} + a_0 \sin x = F. \tag{34}$$

More generally, we can apply the same method to

$$\dddot{x} + \phi_3\ddot{x} + \phi_2\ddot{x} + f(\dot{x}, x) + f_1(x) = F(t), \tag{35}$$

where ϕ_3 and ϕ_2 may both be nonlinear. Introducing the previous notations, Eq. 35 may be written as

$$k + \phi_3 h + \phi_2 g + f(v, x) + f_1(x) = F(t). \tag{36}$$

Equation 30 should then be generalized to

$$\frac{dh}{dt} = k = F(t) - \phi_3 h - \phi_2 g - f(v, x) - f_1(x), \tag{37}$$

and Eq. 31 becomes

$$\frac{dh}{dg} = \frac{k}{h} = \frac{F(t) - \phi_3 h - \phi_2 g - f(v, x) - f_1(x)}{h}. \tag{38}$$

Similar changes should be made in Eqs. 32 and 33 based upon Eq. 36.

INTEGRATION AND INTEGRAL CURVES

As the $v - x$ trajectory is sometimes called an energy curve or an integral curve, the $g - v$ trajectory may be called an energy curve or an integral curve of the second order. Similarly, the $h - g$ trajectory may be called an energy or integral curve of the third order. In general, it may be suggested that a trajectory plotting the $(n - 1)$th derivative *versus* the $(n - 2)$th derivative may be termed an energy or integral curve of the $(n - 1)$th order, where n is the order of the differential equation. Thus, for a second-order differential equation ($n = 2$), we plot the $(n - 1)$th or the first derivative ($v = dx/dt$) *versus* the displacement (x), and the $v - x$ trajectory is an integral or energy curve of the $(n - 1)$th or first order as previously suggested.

Suppose from physical experiments, we get a certain $g - t$ curve, which is the instantaneous acceleration curve. Now we want to find the $v - t$ curve and the $x - t$ curve. As integration is usually more difficult than differentiation, we may suggest the following procedure. Taking slopes at different points on the $g - t$ curve, we get pairs of values of h and g, where $h = dg/dt$. From

$$\frac{dg}{dv} = \frac{h \text{ (from slope)}}{g \text{ (given)}} \tag{39}$$

we can construct the $g - v$ trajectory. From the $g - v$ trajectory, we get pairs of values of g and v. Then, from

$$\frac{dv}{dx} = \frac{g \text{ (given)}}{v \text{ (from } g - v \text{ plot)}} \tag{40}$$

we can construct the $v - x$ trajectory.

Knowing the $g - t$ curve and the $g - v$ trajectory, the $v - t$ curve can be readily plotted. Similarly, from the $v - t$ curve and the $v - x$ trajectory, the $x - t$ curve can be plotted. Thus we have suggested a new way of getting instantaneous velocity and displacement curves, if we are given the instantaneous acceleration curve. For the electrical case, if an instantaneous di/dt curve is given, we can find i and q as a function of time. As $q = \int i\,dt$, it takes integration to find the charge q, when the current i is known as a function of time. This alternative way suggests that from the $i - t$ curve, we first get di/dt or the slope of the $i - t$ curve. Calling di/dt as g, we get $di/dq = g/i$. Then we can construct the $i - q$ trajectory, and get the $q - t$ curve by taking proper points from the original $i - t$ curve and the $i - q$ trajectory.

CONCLUDING REMARKS

The generalized or high order phase-plane method extends the acceleration-plane method (1, 2, 3) to the solution of third and higher order nonlinear differential equations. The methods suggest that a trajectory of the $(n - 1)$th order be first constructed, when the original differential equation is of the nth order. The slope of this trajectory is given by

$$\frac{dx^{(n-1)}}{dx^{(n-2)}} = \frac{x^{(n)}}{x^{(n-1)}}. \tag{41}$$

For a second-order differential equation, this is simplified to

$$\frac{dv}{dx} = \frac{g}{v} \tag{42}$$

since $x^{(2)} = d^2x/dt^2 = g$, $x^{(1)} = dx/dt = v$, and $x^{(0)} = x$, according to the notations previously suggested.

From Eq. 41, we proceed to construct a trajectory of the $(n - 2)$th order by

$$\frac{dx^{(n-2)}}{dx^{(n-3)}} = \frac{x^{(n-1)}}{x^{(n-2)}}. \tag{43}$$

This process is repeated until Eq. 42 is obtained, when we get a trajectory of the first order.

The instantaneous curves for $x - t$ and $v - t$ are first plotted. Then the instantaneous curve for a higher order derivative can be obtained by combining the lower order instantaneous curve, say, the $v - t$ curve, and its higher order trajectory, say, the $g - v$ trajectory. The process is repeated until we get the instantaneous curve of the highest derivative.

The proposed method depends not only on the correlation of the high derivatives but also on the physical or boundary conditions. As time is implicit in all orders of phase-plane plots, a time function $F(t)$ or a time-varying parameter does not offer any additional difficulties to the solution (1, 2, 3).

Acknowledgment

The author records with deep gratitude his ever indebtedness to his mother for constant guidance and inspiration even from far away. This paper is hereby solemnly dedicated to her loving memory, as the sad news of her passing on February 22, 1953 at the age of 71 came on the very day when this paper was completed.

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THE THERMODYNAMICS OF CRITICAL PHENOMENA IN GASES

BY

J. F. LEE [1]

ABSTRACT

Sonic velocity measurements have been used for some time to determine the specific heats of gases at low frequencies following the suggestion of Einstein (1)[2] that the reaction rates of reactive gas mixtures could be found from similar measurements. However, the equation for sonic velocity expressed in terms of the ratio of the specific heats and the isothermal bulk modulus becomes indeterminate when applied to the critical point. In this paper a determinate expression for the sonic velocity at the critical point is derived which permits a direct solution for the constant volume specific heat as a function of the critical values of temperature, specific volume, sonic velocity and Joule-Thomson coefficient. The calculated values obtained from this expression are examined in comparison with existing data. Finally the approach to the problem from the standpoint of classical thermodynamics is appraised in the light of some findings from statistical thermodynamics.

DERIVATION OF C_v AT CRITICAL POINT

The sonic velocity at zero frequency is expressed as follows:

$$a^2 = -\frac{v^2}{m}\left(\frac{\partial p}{\partial v}\right)_s = -\frac{v^2}{m}\frac{C_p}{C_v}\left(\frac{\partial p}{\partial v}\right)_T \tag{1}$$

where a = sonic velocity, m = molecular weight, v = molar volume, and $-v(\partial p/\partial v)_s$ and $-v(\partial p/\partial v)_T$ are the isentropic and isothermal bulk moduli, respectively. It can be shown that for any pure substance,

$$C_p = C_v - T(\partial p/\partial T)_v^2/(\partial p/\partial v)_T.$$

If this expression is substituted in Eq. 1, the following equation is obtained for the sonic velocity.

$$a^2 = \frac{Tv^2}{mC_v}\left(\frac{\partial p}{\partial T}\right)_v^2 - \frac{v^2}{m}\left(\frac{\partial p}{\partial v}\right)_T. \tag{2}$$

At the critical point $(\partial p/\partial v)_T = 0$ and Eq. 2 reduces to the determinate form,

$$a_c^2 = \frac{Tv^2}{mC_v}\left(\frac{\partial p}{\partial T}\right)_v^2, \tag{3}$$

where a_c is the sonic velocity at the critical point.

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[2] The boldface numbers in parentheses refer to the references appended to this paper.

Now the Joule-Thomson coefficient is defined as

$$\mu = \left(\frac{\partial T}{\partial p}\right)_h \tag{4}$$

which, with the aid of the second law of thermodynamics, can be shown to be equivalent to

$$\mu = \frac{1}{C_p}\left[T\left(\frac{\partial v}{\partial T}\right)_p - v\right]. \tag{5}$$

The well known general thermodynamic relations,

$$-\left(\frac{\partial v}{\partial T}\right)_p = \frac{(\partial p/\partial T)_v}{(\partial p/\partial v)_T}$$

and

$$C_p = C_v + T\left(\frac{\partial v}{\partial T}\right)_p\left(\frac{\partial p}{\partial T}\right)_v$$

when substituted in Eq. 5 yield, after simplification, the following equivalent expression for the Joule-Thomson coefficient.

$$\mu = \frac{v\left(\frac{\partial p}{\partial v}\right)_T + T\left(\frac{\partial p}{\partial T}\right)_v}{T\left(\frac{\partial p}{\partial T}\right)_v^2 - C_v\left(\frac{\partial p}{\partial v}\right)_T}. \tag{6}$$

Since $(\partial p/\partial v)_T = 0$ at the critical point, Eq. 6 reduces to

$$\mu_c = \frac{1}{(\partial p/\partial T)_v} \tag{7}$$

where μ_c = the Joule-Thomson coefficient at the critical point.

When Eq. 7 is substituted in Eq. 3, the following expression for the constant volume specific heat at the critical point is obtained.

$$C_v = \frac{T}{m}\left(\frac{v}{a_c\mu_c}\right)^2. \tag{8}$$

For the purpose of comparison, some calculated values of the constant volume molar specific heats are listed with the zero pressure specific heats in the following table. Data for the calculations and for the zero pressure specific heats were obtained from several sources (2–7).

The calculated values appear to be reasonable when compared with the zero-pressure values of the specific heats and compare favorably with the values obtained from extrapolated *P-V-T* data. The calcu-

lated values at the critical point may be used to permit interpolation for intermediate values between the critical point and those points for

Constant Volume Molar Specific Heats at the Critical Point and at Zero Pressure.

Gas	C_v (from Eq. 8) Cal/mole deg.	C_{v_∞} Cal/mole deg.
N_2	11.3	4.98
CO_2	17.4	6.83
O_2	13.1	5.11

which experimental data are available. It is believed that this is a more satisfactory method than that of simply extrapolating experimental specific heats to the critical point.

APPRAISAL OF CALCULATED VALUES OF C_v AT THE CRITICAL POINT

It is desirable, of course, to compare the calculated and calorimetric values of the specific heats at the critical point, but unfortunately published values of the specific heats at the critical point are scarce. However, a calorimetric specific heat of approximately 50 calories per mole degree has been reported for CO_2 (8). The difference between the calorimetric and calculated specific heats is to be expected in view of the limitations implied by the use of Eq. 8. This difference, it is believed, is of the correct order of magnitude to be accounted for by several factors not anticipated by the classical thermodynamic approach employed thus far.

The sonic velocity employed in Eq. 8 should be the thermodynamic value, that is, complete equilibrium must be attained. There is every reason to believe that the measured value of the sonic velocity is not the thermodynamic value because of the large sound attenuation at the critical point due to (1) scattering or dispersion of the sound waves, (2) the increased absorption of high frequency sound waves at the critical point, and (3) configurational relaxations.

Several investigators (9–12) have produced convincing evidence of vibrational heat-capacity lag due to dispersion and high absorption of sound waves when a gas is rapidly compressed. Other investigators (13–15) have established the existence of rotational heat-capacity lag. It is highly probable that when a gas is rapidly condensed the extensive clustering which ensues is affected by the sound waves causing a redistribution of the molecules in the clusters and/or a redistribution of the clusters accompanied by a configurational relaxation process. It is estimated from the previously mentioned publications that a total relaxation time of the order of 10^{-6} second is required.

From the foregoing discussion it must be concluded that $(\partial p/\partial v)_T$ and $(\partial p/\partial T)_v$ are dependent on time. Therefore the system behaves

as though it were only slightly compressible until sufficient time has elapsed for equilibrium to be established.

A subsequent paper will give a quantitative analysis of the constant volume specific heats at the critical point obtained from classical and statistical thermodynamic considerations.

Acknowledgment

The author is indebted to Professor J. S. Doolittle of North Carolina State College for valuable criticism and suggestions.

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ON THE INVERSION OF CONTINUANT MATRICES*

BY

W. J. BERGER[1] AND EDWARD SAIBEL[1]

INTRODUCTION

A square matrix which consists of zero elements everywhere except in the main diagonal and in the diagonals just above and below the main one is called a continuant matrix (1).[2]

The importance of the continuant matrix in physical problems has been pointed out elsewhere (2) and it is the object of this paper to find its inverse. This is done by elementary methods and since the results turn out to be expressible in simple and compact form it is deemed worthwhile to note them explicitly. The authors also note a special case when the inverse of the continuant matrix has a symmetry which they have designated by the term "gnomonic." This is described below.

Interest in the inverse of the matrix may arise not only from the solution of a system of linear equations but (*a*) from its connection with finding the Green's function or approximations to the Green's function (3) and (*b*) the use of iteration methods to find eigenvalues (4). The need for the inverse arises in the latter from the fact that iteration methods yield the dominant eigenvalue and frequently it is the eigenvalue of smallest modulus that is required.

GENERAL THEORY

The Inversion Formulae

To simplify the discussion, we illustrate the development with a continuant of fourth order, but the processes are such that we can easily write the formulae for the nth order. Thus, let the given continuant be

$$A = \begin{bmatrix} a_1 & b_1 & 0 & 0 \\ c_2 & a_2 & b_2 & 0 \\ 0 & c_3 & a_3 & b_3 \\ 0 & 0 & c_4 & a_4 \end{bmatrix}$$

Let $A^{-1} = G$, and as usual let $G_{ij} \equiv G(i, j)$ denote the element in the ith row, jth column position in G.

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[2] The boldface numbers in parentheses refer to the references appended to this paper.

To find G, transform A to the unit matrix I by elementary row operations only (4), beginning with the last row; then if the same set of operations is performed on the unit matrix I, the latter will be transformed into G.

Let T denote the product of elementary operations reducing A to triangular form having zeros everywhere above the main diagonal. We find

$$TA = \begin{bmatrix} k_1 & 0 & 0 & 0 \\ c_2 & k_2 & 0 & 0 \\ 0 & c_3 & k_3 & 0 \\ 0 & 0 & c_4 & k_4 \end{bmatrix}$$

where, if n denotes the order of A, and assuming $a_n \neq 0$,

$$\begin{aligned} k_n &= a_n \\ k_{n-1} &= a_{n-1} - (b_{n-1}c_n)/a_n \\ k_{n-2} &= a_{n-2} - (b_{n-2}c_{n-1})/k_{n-1} \\ &\cdot \\ &\cdot \\ &\cdot \\ k_1 &= a_1 - (b_1c_2)/k_2. \end{aligned}$$

In the event that any one of the k's is zero, the inverse of A does not exist. Note it has been assumed above $k_n = a_n \neq 0$.

Similarly

$$B \equiv TI = \begin{bmatrix} 1 & (-b_1/k_2) & (b_1b_2/k_2k_3) & \left(-\dfrac{b_1b_2b_3}{k_2k_3k_4}\right) \\ 0 & 1 & (-b_2/k_3) & (b_2b_3/k_3k_4) \\ 0 & 0 & 1 & (-b_3/k_4) \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

In general:

$$\begin{aligned} &B_{ii} = 1, \quad i = 1, 2, \cdots, n, \\ &B_{ij} = 0 \quad \text{when } i > j, \\ &B_{rs} = (-1)^{r+s} \prod_{t=0}^{s-r-1} \frac{b_{r+t}}{k_{r+1+t}} \quad \text{when } s > r. \end{aligned}$$

If V denotes the product of row operations reducing TA to I,

$$VTA = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \equiv I,$$

and

$$G = VTI = VB = \begin{bmatrix} \frac{1}{k_1} & -\frac{b_1}{k_2}G_{11} & -\frac{b_2}{k_3}G_{12} & -\frac{b_3}{k_4}G_{13} \\ -\frac{c_2}{k_2}G_{11} & \frac{1}{k_2}(1 - c_2G_{12}) & \frac{1}{k_2}\left(-\frac{b_2}{k_3} - c_2G_{13}\right) & \frac{1}{k_2}\left(\frac{b_2b_3}{k_3k_4} - c_2G_{14}\right) \\ -\frac{c_3}{k_3}G_{21} & -\frac{c_3}{k_3}G_{22} & \frac{1}{k_3}(1 - c_3G_{23}) & \frac{1}{k_3}\left(-\frac{b_3}{k_4} - c_3G_{24}\right) \\ -\frac{c_4}{k_4}G_{21} & -\frac{c_4}{k_4}G_{22} & -\frac{c_4}{k_4}G_{23} & \frac{1}{k_4}(1 - c_4G_{24}) \end{bmatrix}$$

From the construction of G we observe that its elements in the general case n are given by:

$$\left.\begin{aligned} G_{11} &= 1/k_1 \\ G(1,j) &= -\frac{b_{j-1}}{k_j}\cdot G(1,j-1) \quad \text{when } j > 1. \\ G(r,i) &= -(c_r/k_r)G(r-1,i) \quad \text{when } i < r. \\ G(r,r) &= \frac{1}{k_r}[1 - c_rG(r-1,r)] \quad \text{when } r > 1. \\ G(r,s) &= \frac{1}{k_r}\left[\frac{G(1,s)}{G(1,r)} - c_rG(r-1,s)\right] \quad \text{when } s > r. \end{aligned}\right\} \tag{1}$$

Note incidentally that $\det A = \prod_{j=1}^{n} k_j$.

Conditions for Gnomonic Symmetry of $G = A^{-1}$

In Fig. 1 the heavy dots represent elements of a fifth-order matrix G, and the lines 1–1, 2–2, etc., may be called *gnomonic lines of type 1*, which may be defined in the same way for a matrix of order n.

Definition: G has gnomonic symmetry of type 1 if every element in G along the gnomonic line R is equal to G_{rr}.

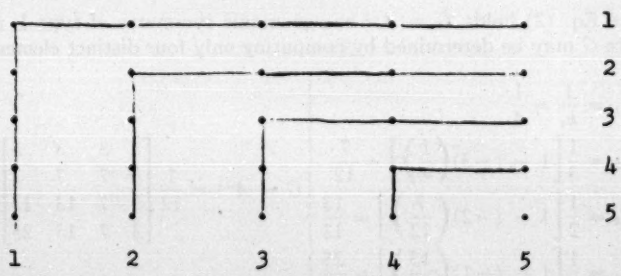

FIG. 1.

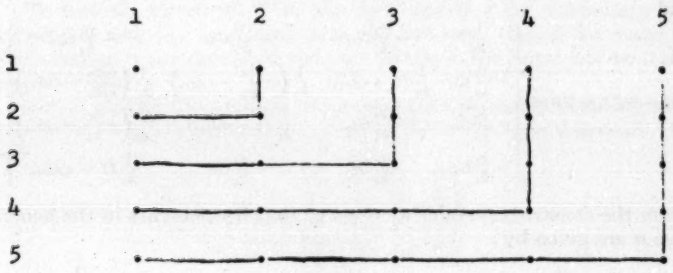

FIG. 2.

In order for $G = A^{-1}$ to be symmetric at all it is necessary that A be symmetric, that is, that $b_j = c_{j+1}$ for $j = 1, 2, \cdots, n-1$. By inspection of the matrix $G = VB$ already given, we easily find that along the type 1 gnomonic line R in G every element will equal G_{RR} if $c_j = -k_j$ for every $j = r+1, r+2, \cdots, n$. Hence G has gnomonic symmetry of type 1 if $b_j = c_{j+1}$ and also

$$c_r = -k_r \quad \text{for all } r = 2, 3, \cdots, n. \tag{2}$$

Example:

$$A = \begin{bmatrix} 7 & -3 & 0 & 0 \\ -3 & 5 & -2 & 0 \\ 0 & -2 & 3 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix}, \quad \begin{cases} k_4 = 1 \\ k_3 = 3 - \dfrac{(-1)^2}{1} = 2 \\ k_2 = 5 - \dfrac{(-2)^2}{2} = 3, \\ k_1 = 7 - \dfrac{(-3)^2}{3} = 4 \end{cases} \quad \begin{cases} c_4 = -1 \\ c_3 = -2 \\ c_2 = -3 \end{cases}$$

Since Eq. (2) holds, $G = A^{-1}$ has gnomonic symmetry of type 1, and hence G may be determined by computing only four distinct elements:

$$\begin{aligned} G_{11} &= \frac{1}{k_1} = \frac{1}{4} \\ G_{22} &= \frac{1}{3}\left[1 - (-3)\left(\frac{1}{4}\right)\right] = \frac{7}{12} \\ G_{33} &= \frac{1}{2}\left[1 - (-2)\left(\frac{7}{12}\right)\right] = \frac{13}{12} \\ G_{44} &= \frac{1}{1}\left[1 - (-1)\left(\frac{13}{12}\right)\right] = \frac{25}{12} \end{aligned} \qquad G = A^{-1} = \frac{1}{12}\begin{bmatrix} 3 & 3 & 3 & 3 \\ 3 & 7 & 7 & 7 \\ 3 & 7 & 13 & 13 \\ 3 & 7 & 13 & 25 \end{bmatrix}.$$

Likewise, as in Fig. 2, gnomonic lines of type 2 may be defined, and one may show that G has gnomonic symmetry of type 2 if

$$\left.\begin{aligned} b_j &= c_{j+1} \quad \text{for } j = 1, 2, \cdots, n-1, \\ \prod_{j=1}^{r} k_j &= (-1)^{r+1}(b_{r-1} + k_r) \prod_{i=1}^{r-1} b_i \quad \text{for } r = 2, 3, \cdots, n. \end{aligned}\right\} \tag{3}$$

Or, alternatively, one may reflect the given matrix A about a line normal to the main diagonal, thereby converting type-2 lines into type-1, so that one may then apply the simpler test (2).

It is also interesting to note that in the gnomonic case the sum of the elements in every row other than the first one must be zero, and that the sum of the elements in the first row gives a very easy evaluation, or check on the computation of k_1 since

$$a_1 + b_1 = k_1.$$

A physical instance of the gnomonic case arises in the torsional vibrations of a shaft (4) where one end is fixed and the other free, since applying a unit torque at some point produces an angular displacement which is constant from the applied point to the free end of the shaft.

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The Atom Age: From Boat-in-Bottle to Sub-in-Sphere.—A monstrous steel ball, 225 feet in diameter, will soon bulge out of the landscape near West Milton, N. Y. Already under construction, the giant sphere is to house a land-based prototype of an atomic power plant for Navy submarines. Its purpose is to protect the surrounding population from radioactive materials should all the reactor controls fail simultaneously—a slim chance, but worth guarding against. The sphere will be supported by a ring of 26 steel columns and will rest in a saucer-like foundation made of reinforced concrete. When completed, a prototype submarine hull will be placed inside and partly immersed in water. Inside the hull will be an atomic power plant representing one of two basic approaches to controlled atomic energy. It will use liquid sodium metal to transfer heat from the reactor core to a "boiler." Water in the boiler will be converted to steam, and the steam will drive the turbines that propel the submarine.

Gas Turbines Used for Oil Wells.—Pumping gas from an oil well back into an oil field reservoir to conserve energy and increase ultimate production of the field will soon be added to the growing list of jobs done by General Electric gas turbines.

J. P. Keller, general manager of the Gas Turbine Department, General Electric Company, announced that the first of ten gas turbines was recently shipped from the factory. They will be used by the Creole Petroleum Corporation in their Tia Juana field located in Lake Maracaibo, Venezuela.

The producing fields in Lake Maracaibo, one of the major oil-producing areas of the world, are dotted with flow stations which receive a mixture of gas and oil from the wells in the lake. The ten gas turbines will be mounted on a platform built on concrete piles about seven miles out from the shore. The platform will be approximately 340 ft. long and 130 ft. wide.

Gas from the flow stations will be piped to the turbine platform, at a pressure of about 10 psi. Ten per cent of the gas will be used as fuel for the turbines.

The turbines will operate centrifugal compressors which will return the remainder of the gas to the wells at a pressure approaching 2000 psi.

Gas turbines were selected because they weigh only one-third as much as reciprocating engines of similar capacity. The small vibration of the rotating type machine compared to reciprocating type was an important factor which made it possible to simplify the platform on which they are mounted.

Turbines are the two-shaft simple cycle type rated at 6000 hp. each. The first two units were shipped in May, and the last unit will be shipped in September. International General Electric Co., G. E. division which is responsible for the company's overseas operations, expects all ten units to be in operation by late July 1954.

The gas turbine is one of the newest entries in the field of prime movers, or sources of power from the products of combustion. Although General Electric engineers have studied the machines and considered their applications for many years, commercial manufacture has been in progress only for the past three years.

Other jobs now being done by gas turbines include central station power generation, natural gas pipeline pumping, locomotive propulsion, and some phases of oil refining.

THE PRINCIPLE OF COMPLEMENTARY ENERGY IN NONLINEAR ELASTICITY THEORY

BY

HENRY L. LANGHAAR [1]

INTRODUCTION

In 1889, Engesser (1)[2] generalized Castigliano's principle of least work to embrace systems with nonlinear characteristics. Engesser's fundamental idea was the substitution of a function called "complementary energy" for the strain energy. This idea was elaborated and illustrated by H. M. Westergaard (2) in 1942. In the last decade, it has received attention in connection with variational principles of plasticity (3). Some theories of plasticity employ the assumption that the work performed on a body depends only on the final state of deformation, and not on the intermediate states, provided that the loads increase proportionately and monotonically during the process of deformation. These theories do not distinguish between anelasticity and nonlinear elasticity, unless unloading occurs.

The theories of Engesser, Westergaard, and Greenberg emphasize the relation of the law of complementary energy to plasticity theory, but they do not show how it is related to the general principles of mechanics. Since the principle of virtual work, $\delta W \equiv 0$, governs the equilibrium of any mechanical system for which infinitesimal displacements are not restricted as to sense, all variational principles of equilibrium stem from this law. The following theory shows that the Legendre transformation is the mathematical device that bridges the gap between the principle of virtual work and the principle of complementary energy.

The forces on a mechanical system may be separated into two sets—the internal forces (forces that are reacted on other parts of the system), and the external forces (forces that are reacted on bodies outside of the system). Accordingly, the principle of virtual work assumes the form, $\delta W' + \delta W'' \equiv 0$, where $\delta W'$ is the virtual work of the internal forces, and $\delta W''$ is the virtual work of the external forces.

A mechanical system with conservative internal forces is said to be "elastic." This definition of elasticity does not imply a linear stress-strain relation, nor does it imply isotropy or homogeneity of the material. If an elastic system passes from a fixed point P_0 to a variable point P in its configuration space, the work W' of the internal forces is

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[2] The boldface numbers in parentheses refer to the references appended to this paper.

independent of the path; hence, it is a function of the terminal configurations, P_0 and P. The initial configuration P_0 is irrelevant, since it merely affects an additive constant in W'. Therefore, $W' = -U(P)$. The function $U(P)$ is called "strain energy" or "potential energy of the internal forces."

SYSTEMS WITH FINITE DEGREES OF FREEDOM

A holonomic elastic system with n generalized coordinates $(x_1, x_2, \cdots, x_n)$ is considered. If the system receives a virtual displacement δx_i, the virtual work of the internal forces is[3]

$$\delta W' = -\frac{\partial U}{\partial x_i}\delta x_i \tag{1}$$

where $U(x_1, x_2, \cdots, x_n)$ is the strain energy.

The virtual work $\delta W''$ of the external forces depends linearly on δx_i; that is,

$$\delta W'' = P_i \delta x_i. \tag{2}$$

The coefficients P_i are the components of a covariant vector in configuration space, called the "generalized external force." This vector is determined by the agencies that cause the external loads. Equation 2 serves for the identification of the physical quantities that are represented by the P's. For example, if x_i represents a component of displacement (or a component of rotation) of a joint of a frame, and if all the external loads are applied at the joints, P_i represents the external force component (or the external moment component) that acts at the joint in the direction x_i. This interpretation is independent of the resistive nature of the material.

The values of the coordinates x_i that correspond to given values of the P's are determined by Eqs. 1 and 2, in conjunction with the principle of virtual work, $\delta W' + \delta W'' \equiv 0$. These equations yield

$$\frac{\partial U}{\partial x_i} = P_i. \tag{3}$$

Equation 3 is conjugate to Castigliano's theorem, for it states that the derivative of the strain energy with respect to a displacement component equals the corresponding force component.

Equation 3 may be solved for the x's, provided that the Hessian determinant of U is different from zero. If certain members of the system can be moved without altering the strain energy, the Hessian vanishes identically. For example, if $\frac{\partial U}{\partial x_1} = 0$, then $\frac{\partial^2 U}{\partial x_1 \partial x_j} = 0$, and therefore $\left|\frac{\partial^2 U}{\partial x_i \partial x_j}\right| = 0$. Accordingly, if U is independent of x_1, Eq. 3

[3] The summation convention for indexes is used.

cannot be solved for the x's; in fact, Eq. 3 then shows that equilibrium is impossible, unless $P_1 = 0$. In this case, x_1 may be regarded as the coordinate of a "loose part" that is free from external force; hence, the configuration of the part is indeterminate. This condition prevails whenever rigid displacements of the entire system are possible, if some of the x's locate the system in space, since a system that is subjected to self-equilibrating forces and moments does not have a unique location in space.

If the Hessian of U is different from zero, Eq. 3 yields

$$x_i = x_i(P_1, P_2, \cdots, P_n). \tag{4}$$

Accordingly, U may be regarded as a function of the P's.

Legendre showed that equations of the type of Eq. 3, with a non-vanishing Hessian, may be transformed by the introduction of a new function $\Upsilon(P_1, P_2, \cdots, P_n)$, defined by

$$\Upsilon = P_k x_k - U. \tag{5}$$

With the present physical interpretations of the variables, the function Υ is called "complementary energy." Differentiation of Eq. 5 yields

$$\frac{\partial \Upsilon}{\partial P_i} = \delta_i{}^k x_k + P_k \frac{\partial x_k}{\partial P_i} - \frac{\partial U}{\partial P_i}$$

or

$$\frac{\partial \Upsilon}{\partial P_i} = x_i + \left(P_k - \frac{\partial U}{\partial x_k}\right)\frac{\partial x_k}{\partial P_i}.$$

Consequently, by Eq. 3,

$$\frac{\partial \Upsilon}{\partial P_i} = x_i. \tag{6}$$

Equation 6 is the law of complementary energy for a system with a finite number of degrees of freedom.

Castigliano's theorem for a system with finite degrees of freedom is a special case of Eq. 6. For the cases treated by Castigliano, U is a homogeneous quadratic function of the x's; that is,

$$U = \tfrac{1}{2} a_{ij} x_i x_j. \tag{7}$$

Consequently, Eq. 3 yields $a_{ij}x_j = P_i$. With Eq. 7, this yields $U = \frac{1}{2}P_k x_k$. Then, by Eq. 5, $U = \Upsilon$. Thus, if U is a homogeneous quadratic function of the x's, Eq. 6 yields

$$\frac{\partial U}{\partial P_i} = x_i. \tag{8}$$

This is Castigliano's theorem.

CONSTRAINING FORCES

To calculate the external constraining forces on a system, we "cut" certain external constraints, and augment the generalized external force P_i by components $(Q_1, Q_2, \cdots, Q_r)$ that represent the forces exerted by the deleted constraints. The cutting of constraints increases the number of degrees of freedom of the system; therefore, the coordinates x_i are augmented by terms $(y_1, y_2, \cdots, y_r)$. The physical meanings of the quantities Q_α are disclosed by the relation

$$\delta W''' = P_i \delta x_i + Q_\alpha \delta y_\alpha$$

where $\delta W'''$ now represents the virtual work of the external forces P_i and the constraining forces Q_α.

For the "cut" system, Υ is a function of the variables $(P_1, \cdots, P_n; Q_1, \cdots, Q_r)$. Equation 6 remains valid, with the supplementary equations,

$$\frac{\partial \Upsilon}{\partial Q_\alpha} = y_\alpha,$$

provided that the cutting of constraints does not cause the Hessian of U to vanish. The requirement that the Hessian shall not vanish ordinarily means that no constraints may be removed from a statically determinate structure.

Because of the actual constraints, the supplemental coordinates y_α are zero. Consequently,

$$\frac{\partial \Upsilon}{\partial Q_\alpha} = 0. \tag{9}$$

Equation 9 determines the "redundant constraining forces" Q_α as functions of the external loads P_i. Equation 9 is a generalization of Castigliano's principle of least work.

COMPLEMENTARY ENERGY OF ELASTIC CONTINUA

Equations 5 and 6 are valid for an elastic body that is loaded by a finite number of point forces, provided that the constraints preclude rigid displacements of the body. The quantities P_i may be interpreted to be the magnitudes of the concentrated loads. The quantities x_i are then the deflections of the loads P_i in the directions of the loads. Equation 5 serves to verify that a function Υ exists, such that the deflections are determined by Eq. 6. In problems of large deflections, the actual determination of Υ may be very difficult, since Eq. 5 is of no avail unless the quantities x_i and U are expressed as functions of the P's.

A significant simplification results, if the geometrical approximations of small-deflection theory are used. Then there exists a complementary

energy density function Ω, such that the complementary energy of an elastic structure is the volume integral of Ω. Consequently, if the deflections of a structure are so small that the geometry of the structure is appreciably unchanged, the complementary energies of the individual members of the structure are additive. The proof of these statements follows.

In the small-deflection theory of elasticity, the strain tensor is

$$\epsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \tag{10}$$

where (x_1, x_2, x_3) are rectangular coordinates, and (u_1, u_2, u_3) are the components of the displacement vector. The stress tensor is denoted by σ_{ij}. The following alternative notations will be used:

$$\sigma_1 = \sigma_{11}, \quad \sigma_2 = \sigma_{22}, \quad \sigma_3 = \sigma_{33}, \quad \sigma_4 = \sigma_{23}, \quad \sigma_5 = \sigma_{31}, \quad \sigma_6 = \sigma_{12}$$
$$\epsilon_1 = \epsilon_{11}, \quad \epsilon_2 = \epsilon_{22}, \quad \epsilon_3 = \epsilon_{33}, \quad \epsilon_4 = 2\epsilon_{23}, \quad \epsilon_5 = 2\epsilon_{31}, \quad \epsilon_6 = 2\epsilon_{12}.$$

For an elastic material, there exists a strain-energy-density function $W(\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4, \epsilon_5, \epsilon_6)$. If the material is anisotropic, the function W is not invariant under a rotation of axes.

It is shown in the theory of elasticity (4) that the stress-strain relations are

$$\sigma_\alpha = \frac{\partial W}{\partial \epsilon_\alpha}. \tag{11}$$

If the Hessian of W does not vanish, Eq. 11 implies that the strain components are functions of the stress components. Then W may be regarded as a function of the σ's. The complementary energy density function $\Omega(\sigma_1, \sigma_2, \cdots, \sigma_6)$ is defined by

$$\Omega = \epsilon_\alpha \sigma_\alpha - W. \tag{12}$$

The Legendre transformation now yields

$$\frac{\partial \Omega}{\partial \sigma_\alpha} = \epsilon_\alpha. \tag{13}$$

Equation 13 is conjugate to Eq. 11.

If the stress-strain relation of the material is linear, W is a homogeneous quadratic function of the ϵ's. Then the same reasoning that leads to Eq. 8 yields $W = \Omega$; that is, if the stress-strain relation is linear, the complementary energy density is identical to the strain-energy density. This is true, even though the material is anisotropic.

The complementary energy of an elastic body B is defined by

$$\Upsilon = \int_B \Omega dv, \tag{14}$$

where dv denotes a differential element of volume. Equation 14 leads to known formulas for the complementary energies of prismatic members that are subjected to bending, twisting, and direct tension. For example, if a rod sustains a constant tensile stress, Eq. 14 yields

$$\Upsilon = Fe - U \tag{15}$$

where F is the tension in the rod and e is the extension of the rod. There is a well known geometrical interpretation of Eq. 15. If F is plotted as a function of e, the area below the curve is U. Consequently, by Eq. 15, Υ is the area above the curve and below the line F = constant. An analogous interpretation applies for a member that is subjected to pure shear.

It is necessary to show that Eq. 14 is consistent with the generalized Castigliano theorem. To this end, we consider an elastic body B that is mounted so that rigid displacements are impossible. The body (which may be multiply connected) is subjected to body forces and to boundary stresses $\sigma_{ij}n_j$, where n_j is the outward-directed unit normal to the surface of the body. If the boundary stresses receive a variation $n_j\delta\sigma_{ij}$, the stresses throughout the body receive a variation $\delta\sigma_{ij}$. Then the variation of Υ is

$$\delta\Upsilon = \int_B \delta\Omega dv = \int_B \frac{\partial\Omega}{\partial\sigma_\alpha}\delta\sigma_\alpha dv.$$

Hence, by Eq. 13,

$$\delta\Upsilon = \int_B \epsilon_\alpha\delta\sigma_\alpha dv = \int_B \epsilon_{ij}\delta\sigma_{ij}dv = \int_B \frac{\partial u_i}{\partial x_j}\delta\sigma_{ij}dv$$

$$= \int_B \frac{\partial}{\partial x_j}(u_i\delta\sigma_{ij})dv - \int_B u_i\frac{\partial}{\partial x_j}(\delta\sigma_{ij})dv.$$

The entire body B must be in equilibrium under the action of the body forces and the boundary stresses. The variation of the boundary stresses is restricted by this condition. Then, the resulting variation of the internal stresses conforms to the differential equations of equilibrium; that is,

$$\frac{\partial}{\partial x_j}(\delta\sigma_{ij}) = 0.$$

Consequently,

$$\delta\Upsilon = \int_B \frac{\partial}{\partial x_j}(u_i\delta\sigma_{ij})dv.$$

The divergence theorem now yields

$$\delta\Upsilon = \int_S u_i n_j \delta\sigma_{ij}\, dS, \tag{16}$$

where S denotes the surface of the body. Equation 16 expresses the variation of Υ in terms of a variation of the boundary stresses that is consistent with the conditions of equilibrium.

Suppose that the boundary S may be divided into two parts—a fixed part S' on which u_i vanishes because of the nature of the constraints, and a deformable part S''. The part S' may consist of a few small spots, of the nature of point supports. Since u_i vanishes in S', the region of integration (Eq. 16) is reduced to S''. The variation of the boundary stresses in S'' is not restricted by the equilibrium condition, since equilibrium may be maintained by compensating reactions in S'.

Consider a high concentration of load on a small area S_0 of the region S''—the type of load that is ordinarily called a "point force." If the variation of the boundary stresses is restricted so that $n_j\delta\sigma_{ij} = 0$ at all points of S'' that do not lie in S_0, Eq. 16 yields

$$\delta\Upsilon = \int_{S_0} u_i n_j \delta\sigma_{ij}\, dS. \tag{16'}$$

If the variation of the boundary stresses is further restricted so that each component of the vector $n_j\delta\sigma_{ij}$ retains a constant sign in S_0, the theorem of the mean for integrals yields

$$\delta\Upsilon = \bar{u}_i \int_{S_0} n_j \delta\sigma_{ij}\, dS,$$

where $\bar{u}_i$ is the value of u_i at some point of S_0. The total load F_i on the small area S_0 is

$$F_i = \int_{S_0} n_j \sigma_{ij}\, dS.$$

Consequently,

$$\delta\Upsilon = \bar{u}_i \delta F_i.$$

Letting the vector δF_i be collinear with the vector F_i, we obtain

$$\delta\Upsilon = w\delta F,$$

where F is the magnitude of the point force F_i, and w is the component of $\bar{u}_i$ in the direction of F_i; that is, w is the deflection of the load F. Also,

$$\delta\Upsilon = \frac{\partial\Upsilon}{\partial F}\,\delta F.$$

Therefore,

$$w = \frac{\partial\Upsilon}{\partial F}. \tag{17}$$

This is the desired generalization of Castigliano's theorem. If the complementary energy of any fixed body or structure is expressed as a

function of the external loads, Eq. 17 determines the deflections at points of load concentration. Since a fictitious load of zero magnitude may be applied anywhere, Eq. 17 determines the deformation of the body completely. It has been proved that the function Υ is determined by Eqs. 12 and 14, if the deflections are so small that the geometry of the structure is appreciably unchanged.

The generalized Castigliano theorem for couples follows from Eq. 17. If two parallel concentrated loads, F_1 and F_2, with opposite senses, act perpendicular to the ends of a straight line segment of length a, Eq. 17 shows that the rotation of the line segment is

$$\alpha = \frac{1}{a}\frac{\partial \Upsilon}{\partial F_1} + \frac{1}{a}\frac{\partial \Upsilon}{\partial F_2}.$$

If F_1 and F_2 are functions of a single variable F,

$$\frac{\partial \Upsilon}{\partial F} = \frac{\partial \Upsilon}{\partial F_1}\frac{\partial F_1}{\partial F} + \frac{\partial \Upsilon}{\partial F_2}\frac{\partial F_2}{\partial F}.$$

In particular, if $F_1 = F_2 = F$, this yields

$$\frac{\partial \Upsilon}{\partial F} = \frac{\partial \Upsilon}{\partial F_1} + \frac{\partial \Upsilon}{\partial F_2}.$$

Consequently,

$$\alpha = \frac{1}{a}\frac{\partial \Upsilon}{\partial F}.$$

Since the equal and opposite forces, F_1 and F_2, constitute a couple of magnitude $M = Fa$, this equation may be written as follows:

$$\alpha = \frac{\partial \Upsilon}{\partial M}. \tag{18}$$

This is the generalized Castigliano theorem for moments. It determines the angular deflection of the arm of any external couple M.

PRINCIPLE OF STATIONARY COMPLEMENTARY ENERGY

Equation 9 may be interpreted to mean that the constraining forces Q_α provide a stationary value to the complementary energy. There is a related theorem in the small-deflection theory of elasticity, concerning bodies with stress-type boundary conditions. The bodies need not be simply connected.

A stress distribution will be said to belong to Class I, if it has the following properties:

V. 19

(*a*) It is continuous with its first and second derivatives.
(*b*) It satisfies the boundary conditions.
(*c*) It satisfies the differential equations of equilibrium.

The theorem of stationary complementary energy asserts that a stress distribution of Class I satisfies the compatibility conditions if, and only if, it provides a stationary value to the complementary energy among stress distributions of Class I. Although this principle is known (3), the following proof differs from those that have been presented before.

The differential equations of equilibrium are

$$\begin{aligned} P &= \frac{\partial \sigma_1}{\partial x} + \frac{\partial \sigma_6}{\partial y} + \frac{\partial \sigma_5}{\partial z} + X = 0 \\ Q &= \frac{\partial \sigma_6}{\partial x} + \frac{\partial \sigma_2}{\partial y} + \frac{\partial \sigma_4}{\partial z} + Y = 0 \\ R &= \frac{\partial \sigma_5}{\partial x} + \frac{\partial \sigma_4}{\partial y} + \frac{\partial \sigma_3}{\partial z} + Z = 0. \end{aligned} \tag{19}$$

Here, the notations (x, y, z) are used instead of (x_1, x_2, x_3). The quantities (X, Y, Z) are the components of body force.

To render the complementary energy T stationary in the class of stresses that satisfy Eq. 19 and the boundary conditions, we multiply the quantities (P, Q, R) by Lagrange multipliers (u, v, w) that are functions of (x, y, z), and add the resulting terms (5) to the integrand in Eq. 14. Thus, a modified complementary energy function $\overline{\mathrm{T}}$ is obtained:

$$\overline{\mathrm{T}} = \iiint (\Omega + uP + vQ + wR)\, dx\, dy\, dz. \tag{20}$$

The integrand in Eq. 20 is denoted by F. The Euler equations for $\overline{\mathrm{T}}$ are

$$\frac{\partial F}{\partial \sigma_\alpha} - \frac{\partial}{\partial x}\frac{\partial F}{\partial (\sigma_\alpha)_x} - \frac{\partial}{\partial y}\frac{\partial F}{\partial (\sigma_\alpha)_y} - \frac{\partial}{\partial z}\frac{\partial F}{\partial (\sigma_\alpha)_z} = 0,$$

where $(\sigma_\alpha)_x$ denotes $\partial \sigma_\alpha / \partial x$, etc. These equations yield

$$\begin{gathered} \frac{\partial \Omega}{\partial \sigma_1} = \frac{\partial u}{\partial x}, \quad \frac{\partial \Omega}{\partial \sigma_2} = \frac{\partial v}{\partial y}, \quad \frac{\partial \Omega}{\partial \sigma_3} = \frac{\partial w}{\partial z} \\ \frac{\partial \Omega}{\partial \sigma_4} = \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}, \quad \frac{\partial \Omega}{\partial \sigma_5} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}, \quad \frac{\partial \Omega}{\partial \sigma_6} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}. \end{gathered} \tag{21}$$

The differential equations that determine the stresses σ_α are obtained by elimination of the Lagrange multipliers (u, v, w) from Eq. 21. This

is accomplished by processes of differentiation, addition, and subtraction, in conjunction with Eq. 19. These steps lead automatically to the compatibility equations, since, by Eq. 13, Eqs. 21 are the strain-displacement relations in the small-displacement theory of strain, if (u, v, w) are interpreted to be the components of the displacement vector. Accordingly, the compatibility equations are the Euler equations of the variational problem. Consequently, the compatibility equations are necessary and sufficient for stationary complementary energy in stress distributions of Class I.

The theorem that has been proved frequently provides a convenient method for deriving the compatibility equations for special types of coordinates or for special types of stress-strain relations, particularly if the equilibrium equations can be eliminated by means of a stress function. As a simple illustration, the complementary energy of an isotropic linearly-elastic plate that is subjected to plane stress is

$$\Upsilon = U = \frac{h}{2E}\int\int [F_{xx}^2 + F_{yy}^2 - 2\nu F_{xx}F_{yy} + 2(1+\nu)F_{xy}^2]\,dx\,dy$$

where F is the Airy stress function. The Euler equation for this integral is $\nabla^4 F = 0$. This is the known compatibility equation of plane elasticity theory.

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Postscript. The author learned belatedly of T. M. Charlton's excellent articles (6, 7). Charlton's treatment discloses the principle of complementary energy from a new perspective, since it employs the law of conservation of energy as a starting point.

ADDENDUM AND CORRECTION

TO

RADIO TECHNOLOGY AND THE THEORY OF NUMBERS*

BY

BALTH. VAN DER POL

It was stated in this lecture that the greatest prime number, known at the time, was recently calculated by Dr. D. H. Lehmer, Los Angeles. This number is $(2^{1279} - 1)$. The number of digits of this prime number, when written in the decimal system, is 386 (not 397 as erroneously given in the paper).

Since the above lecture was given, Dr. Lehmer has succeeded in finding even two greater prime numbers (see: "Math. Tables and Other Aids to Computation," January, 1953, page 72). They are $(2^{2203} - 1)$ and $(2^{2281} - 1)$. In the usual decimal notation these numbers contain 664 and 687 digits, respectively.

* This JOURNAL, Vol. 255, p. 476 (June, 1953).

NOTES FROM THE NATIONAL BUREAU OF STANDARDS*

VERSATILE PHOTOMETRIC GONIOMETER

A goniometer of unusual versatility and precision has been devised by T. H. Projector and associates of the National Bureau of Standards for use in the Bureau's photometric studies of aircraft and other lighting equipment. Designed for use on a 100-ft. photometric range, the NBS goniometer is constructed for maximum rigidity and convenience and in such a way as to minimize errors due to backlash and torsion.

Goniometers used to measure the angular position of a light source on general-purpose photometric ranges must meet severe requirements. Test equipment may vary in weight over a wide range, and the distribution of weight and bulk may be quite variable. Yet a high degree of accuracy is required. The NBS goniometer was specially designed to combine accuracy with flexibility in order to accommodate the many types of photometric measurements made on the Bureau's 100-ft. range and the great variety of equipment studied.

Conventional goniometers of this type ordinarily have two axes of rotation at right angles. Such goniometers may be divided into those having a fixed vertical axis and those having a fixed horizontal axis. Occasionally goniometers are also constructed in which cone angles are measured about one axis and polar angles are measured about the other axis. The new NBS goniometer has three possible axes of rotation and can be used as any one of the three types of goniometers.

Of the three axes of rotation, only one, which measures horizontal angles, is permanently fixed. Of the other two axes, both of which may be rotated, one measures "horizontal" angles while the other measures "vertical" angles. In any given set of measurements, only one of the two horizontal angle rotations is used, the choice being dependent on the desired goniometric geometry or on the mounting requirements of the test equipment.

The horizontal angle rotations are obtained with rotary tables of the type used on machine tools while the vertical angle rotations are obtained by means of two U-frames mounted on the lower table and supporting the upper table. The lower rotary table, 25 in. in diameter, is mounted on a heavy welded base and supports an outer U-frame made of welded iron channels. The second U-frame, constructed from lengths of iron pipe welded together, is mounted within the outer U-frame. This inner frame may be rotated about a horizontal axis extending through ball bearings set on pedestals at the tops of the arms of

* Communicated by the Director.

the outer frame. The upper rotary table, upon which test equipment is usually mounted, is set at the bottom of the inner U-frame. T-slots in the 15-in. surface plate of this table aid in mounting equipment. The scales on the micrometer drives are graduated at 0.1-deg. intervals on the upper rotary table, 0.05-deg. intervals on the lower rotary table, and 0.01-deg. intervals on the drive for the inner U-frame.

The handwheel drive for rotation of the inner U-frame is linked to the frame by a gear reducer, a pinion, and a large sector gear. A pair of fixed counterweights is attached to the ends of tubular shafts on each arm of the inner U-frame to balance the frame about its axis of rotation. In addition, a pair of adjustable weights permits balancing the frame with test equipment mounted in place, and a constant torque is supplied by a small weight suspended from a pulley connected to the axis of the frame.

To balance the inner frame after the test equipment has been mounted on the rotary table, the frame is disengaged from the drive and then freely rotated until the arms are horizontal, in which position the balancing weights can be adjusted easily and balance determined accurately. With the inner frame balanced and the small weight suspended from the pulley, the frame is subjected to a constant torque of about 4 lb.-ft. at all settings of the "vertical" angle.

For most photometric work, the upper rotary table is used for "horizontal" angle measurements. This means that the axis of the "horizontal" angle measurements rotates as the vertical angle setting is changed. However, where it is necessary that the axis of horizontal angle measurements remain fixed, the lower rotary table is allowed to rotate and the other table does not turn on its axis. Rotation of the lower table has also been found useful occasionally when the equipment mounted for study is too bulky to clear the arms of the inner frame.

For cone angle measurements, the inner U-frame is rotated into the position where its arms are horizontal and the beam from the test equipment projects horizontally through the open end of the U-frame toward the photometer. From this position of the inner frame, cone angles are set by rotation of the inner frame, and polar angles are set by rotation of the upper rotary table.

Backlash and variable torque errors have been greatly reduced in the new NBS goniometer by the adjustable balancing arrangement and the constant torque device. In conventional goniometers, which lack balancing arrangements, backlash errors are minimized by rotating the goniometer in one direction only during a traverse. However, when such a goniometer is rotated through a vertical angle, there may be considerable variation in the torque loading. Under these conditions, even when rotation is in one direction only, backlash errors may be large and may vary considerably from test to test.

Although backlash is the most usual source of error in photometric

goniometry, errors due to variable torsion may sometimes occur in parts subjected to torsional stress. Errors of this type have been kept to a minimum in the NBS goniometer by rigid welded construction of the inner frame.

To evaluate the accuracy of the NBS goniometer, angular intervals indicated by the goniometer were compared with those measured by a transit mounted on the upper rotary table in place of a test unit. No error greater than 0.03 deg. in the measurement of angular intervals up to 90° was found with either of the rotary tables. Errors in the measurement of vertical angles with the inner U-frame drive system were found to vary with angular interval from a maximum of 0.02 deg. in a 1-deg. traverse to a maximum of 0.10 deg. in a 90-deg. traverse.

NOTE: For further details, see "Versatile Goniometer for Projection Photometry," by T. H. Projector, *Illuminating Engineering*, Vol. 48, p. 192 (1953).

"CELLULAR" ELECTRONIC CONSTRUCTION

Printed electronic circuits in which conducting patterns etched on plastics take the place of conventional wiring have come into fairly wide use. For the problem of connecting components and tubes to the printed sheets several solutions have been offered. However, the diversity of these solutions and their inability to gain widespread acceptance indicates that the optimum answer has not yet been found.

In a novel approach to the problem currently being investigated by the National Bureau of Standards, small 3-contact molded blocks or cells, each containing one or two circuit elements—resistors, capacitors, inductors—are pressed against the etched circuit pattern by means of springs that are extensions of the tube socket contacts. No soldering is needed. This experimental technique is one of a number under study at the Bureau in a program, sponsored by the Navy Bureau of Aeronautics, for improving construction and maintenance of electronic equipment. Proposed by Dr. P. J. Selgin of the NBS engineering electronics laboratory, the cellular assembly method has several interesting features that could prove advantageous.

The individual molded cells are about $\frac{7}{8}$ in. high by $\frac{1}{2}$ in. wide by $\frac{1}{4}$ in. thick. Each has three contacts, one on the top and two on the bottom. The cells are grouped together in "building blocks," each comprising two tubes and twelve cells held in a compact bundle by means of a suitable frame. The top surface of the block consists of a spring assembly containing the tube sockets and the necessary spring contacts. When the block is fastened to the printed base plate by means of screws, springs in the spring assembly apply substantial pressure to the top terminal of each cell and hold the two bottom terminals firmly in contact with the printed circuit pattern. Positive and noise-free electrical connection is further assured by the application of a thin film of grease to the cell contacts.

The two-tube block is considered an optimum-sized subassembly in the NBS system. Any number of the blocks can be mounted on a suitably-printed base plate of sufficient area. Potentially inexpensive, they are compact (about $2\frac{1}{4}$ by 1 by $1\frac{5}{16}$ in., exclusive of tubes) and are easy to store and to handle. They are extremely rugged, and as long as a block is secured to the base plate none of the cells can vibrate or shake loose.

A noteworthy feature of the technique is the achievement of quick replaceability—of both blocks and cells—without the use of plugs or connectors. If conventional "plug-in" assemblies were made as small as the NBS blocks, the plugs would add substantially to both size and cost. The elimination of both soldering labor and multiple connectors results in a double saving.

In case of trouble, an entire block can be easily removed for repair or replacement, simply by loosening the screws that hold it to the base plate. Either on the spot or after return to the factory or service laboratory, defective blocks can be quickly repaired by replacing faulty cells. Each cell is identified by suitable markings.

An important aspect of the use of three-terminal cells in the NBS technique is the fact that positive pressure can be maintained at three points, and only three points, by a single spring. Fortunately, in the great majority of electronic circuits no more than two circuit elements need be connected electrically to a single tube electrode. A three-terminal cell, therefore, besides being easy to hold under firm spring pressure, will in general provide enough electrical contacts for the elements associated with any tube pin. Exceptional cases can be taken care of in the NBS system by providing for a spare cell not connected to the tube pins. It is also possible to provide for three-terminal cells of double or triple the standard thickness to accommodate occasional oversized elements.

An experimental 9-tube cellular circuit already constructed at the Bureau appears to confirm the practical possibilities of this type of construction. For convenience, the limited number of cells needed were formed at room temperature using a casting resin, although this is not the best material. For quantity production, cells could be molded in phenolic by the process now in wide use for making resistors and capacitors. The components would be spotwelded together and to the terminal tabs before molding.

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The following papers will appear in this JOURNAL within the next few months:

LIVERSIDGE, HORACE: Industry's Responsibility in the Shortage of Engineers.
KLINGER, H. H.: Radio Astronomy.
MOON, PARRY, AND DOMINA EBERLE SPENCER: TEM Waves in Cylindrical Systems.
CUNNINGHAM, W. J.: Amplitude Variations in a Frequency-Modulated Oscillator.
PAI, S. I.: On Turbulent Flow in Circular Pipe.
BROOKS, H. B.: Electrical Indicating Instruments Used in Early Edison Central Stations.
BLASCHKE, ALFRED C., AND CRAIG L. TAYLOR: The Mechanical Design of Muscle-Operated Arm Protheses.
SKILLING, HUGH, AND JOHN C. BECKETT: Control of Air Ion Density in Rooms.

NOTES FROM THE FRANKLIN INSTITUTE MUSEUM

A. C. CARLTON, DIRECTOR

FRANKLINIANA

Students of Franklin's life and work frequently express disappointment at being unable to find any considerable collection of material associated with the great man. So widespread has been the interest displayed in Benjamin Franklin that any object associated with his work has been keenly sought, and what has survived is now widely dispersed. The collection of Frankliniana in the Museum assumes increasing importance as it slowly accumulates, and it gives cause for satisfaction that it is representative of the many-sided Franklin's interests.

The latest addition to the collection is a writing desk intimately associated with Franklin's later life. The desk is tall and narrow, of a design that is not often seen. It is constructed of rosewood, with an inlaid sunburst of satinwood on the outside of the leaf dropped when the desk is in use. At the bottom are three small drawers and the top portion is fitted with shelves behind a wire grill. The desk was purchased by Franklin during his residence in France and was among the household effects he brought to this country. Later, it was given to Benjamin Franklin Bache, the grandson who accompanied the statesman to France and who eventually became his secretary. The desk has remained in the family and has been lent to the Museum by the Misses Margaret and Emily Bache, of Philadelphia, who have previously enriched the Museum's collection of Frankliniana. On the shelves are displayed a collection of old china plates illustrating some well known phrases taken from Franklin's writings. The group of plates was also lent by the Misses Bache.

Franklin was such a voluminous writer that it is not inappropriate to have two of the desks at which he must have written many letters, for, it will be recalled, the Museum's collection already includes the lap-desk he used while travelling.

One of the most interesting features about this collection of Frankliniana is that it is representative of many aspects of this versatile man. There are several examples of his scientific work, most conspicuous of which is the friction machine employed for the production of static electricity in the experiments which brought him universal fame. This is at once so simple and efficient when compared with its contemporary devices made for the same purpose, that one concludes it must have been constructed from his own design. For it is characteristic of all that Franklin did that he did it in a simpler and more direct way than other men.

Then, too, are the printing tools he used, reminding us of the occupation he followed with such diligence that it provided Franklin with the means for an early retirement and afforded him the leisure to pursue his scientific investigations and to devote himself to public service.

Another device, an odometer, brings to mind his work as Postmaster, for this was attached to the axle of his carriage when he made his

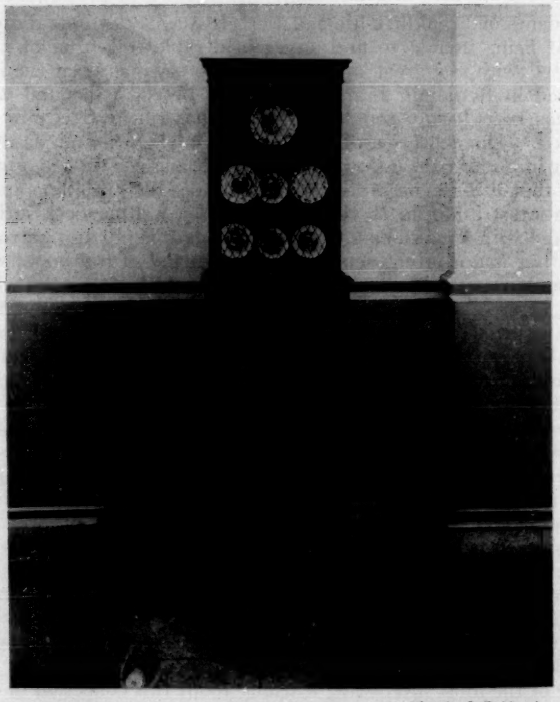


Photo by J. E. Marsden

Franklin's desk.

journeys to inspect postoffices. By means of this odometer he was able to make a more accurate assessment of the fees to be paid for postage, which was then based on mileage, while marking the roads with milestones for the benefit of other travellers.

No collection of Frankliniana would be complete without representative pieces of private life, and the Museum is fortunate in the posses-

sion of several. Franklin was endowed with a wide circle of friends who benefitted from his conversation, sprinkled with homely wisdom and salty wit. The visitor who looks upon the tea-service given to Franklin by his dear friend Madame Helvetius cannot refrain from speculating upon the sparkling talk that must have taken place around the table on which these cups were set.

Unique among the precious possessions displayed in the Franklin Memorial Hall is the court sword, on which is engraved the stern admonition "Draw me not save with honor." One would hardly think that Franklin required any such exhortation. He was more familiar with the use of the pen and, we are told, the pen is mightier than the sword. Franklin never needed any reminder that he must exercise none but honorable intentions when he employed his pen. In the great volume of writing that came from his pen there is not a line that needed to be withdrawn, not a barb that caused pain, nor ever a reflection that caused the writer embarrassment.

T. C.

NOTES FROM THE BARTOL RESEARCH FOUNDATION

W. F. G. SWANN, DIRECTOR

GENERAL STATEMENT

BY

W. F. G. SWANN

The laboratory has continued its investigations on the lines of nuclear physics, radioactivity, cosmic radiation, thermionics, and the solid state. The details of some of the more recent findings will be cited below.

The Director has continued his collaboration with the Institute's Laboratories for Research and Development, and particularly with Dr. W. Philippoff, for whose experimental findings he has evolved a mathematical theory correlating the significant elements of the said findings.

In connection with solid state work at the Bartol Research Foundation and, in particular, with certain lectures on statistical mechanics, the Director has evolved certain new methods of approach in the form of statistical theorems in relation to thermodynamics.

Dr. C. E. Mandeville has collaborated with the Biochemical Research Foundation in certain phases of the work of that Foundation pertaining to the stimulation of ultraviolet light by X-rays.

Since the last notes were submitted, the following papers have appeared:

"The Total Neutron Cross-Sections of Gold, Chlorine and Phosphorus," by S. C. Snowdon and W. D. Whitehead.

"Specific Primary Ionization of H_2, He, Ne, and Al in High Energy Electrons," by G. W. McClure.

"Fundamentals in the Behavior of Electrets," by W. F. G. Swann.

"The Phosphorescence of Thoria," by C. E. Mandeville and H. O. Albrecht.

In addition, C. E. Mandeville, F. R. Metzger, W. E. Danforth, J. F. Marshall and G. W. McClure have presented papers before various learned societies.

The Director has completed a paper on cosmic rays requested by *Sky and Telescope*. He has also addressed the Frankford Arsenal and the Society for Industrial and Applied Mathematics.

COLOR PHENOMENA IN THORIUM OXIDE*

BY

W. E. DANFORTH

Studies have been made of the causes of various color forms which thorium oxide is found to assume. The material itself is of practical interest as a cathode emitter and these studies were part of a program concerned with the electrical properties of compounds at high temperatures.

Specimens of crystalline fused thorium oxide may exist in the following colors: red, water white, yellow, and black. It was at first supposed that the coloration was the result of various impurities. We have ascertained that the several forms are related to the following circumstances: (1) oxygen content, (2) heat treatment, (3) radiation exposure.

When first removed from the fusing furnace, the material is of a deep red color. We have termed this *oxygen-red* since it is found that removal of a very small amount of oxygen by vacuum firing at 1000° C. will eliminate the coloration and leave a water-white crystal. The amount of oxygen responsible for the change is less than 10^{17} per cm.[3] The white crystal may be restored to its original red color by exposure to oxygen at 1000° C. However, the white crystal may also be made red (with very similar absorption spectrum) by exposure to ultraviolet radiation, for example, sunlight. This is termed *radiation-red*. It differs markedly from oxygen-red by the fact that merely heating to 200° C. bleaches the crystal to the white form.

The above forms are all members of what we call the *high-oxygen* state of thorium oxide. If a high-oxygen crystal is vacuum fired at 1300° C. or higher, a weighable amount (1 part in 10^5) of oxygen is evolved. The resulting *low-oxygen* crystal is a lemon yellow color, has a marked shift of the ultraviolet absorption edge, but gives the same X-ray pattern. It is susceptible to radiation reddening to a much smaller degree than is the *white high-oxygen* form.

Presumably, the *low-oxygen* state is the more nearly of stoichiometric proportions. In thermionic application, this state is the only one involved.

GAMMA-GAMMA ANGULAR CORRELATION IN SE^{76}†

BY

F. R. METZGER AND W. B. TODD

In the course of a general investigation of the angular correlations of the Se^{76} gamma-rays, the 0.55–0.65 Mev cascade has been studied, using pulse height selection in both channels.

* Assisted by the Office of Naval Research.
† Assisted by the joint program of the Office of Naval Research and the Atomic Energy Commission.

The anisotropy found with pulse height selection is twice as large as the one reported for the bulk correlation by Kraushaar and Goldhaber (*Phys. Rev.*, **89**, 1081, (1953)]. The spin assignment 2-2-0 is confirmed. The 0.65 Mev transition is almost pure quadrupole, the admixture of magnetic dipole being smaller than 15 per cent. The final analysis will be possible following the investigation of the other cascades in Se^{76}.

ANGULAR DISTRIBUTION OF NEUTRONS SCATTERED FROM ALUMINUM, IRON AND LEAD*

BY

W. D. WHITEHEAD AND S. C. SNOWDON

The differential cross-sections for the scattering of 3.7 Mev neutrons by Al, Fe, and Pb have been measured between 13° and 140° with an angular resolution of ± 10 degrees. The measurements were performed in a ring geometry in which the angle variation was accomplished by an axial movement of the ring. Corrections for the higher order scattering have been determined experimentally by varying the thickness of the scatterer and extrapolating to zero thickness. Since the energy response of the neutron detector (Zn-s lucite button mounted on a photomultiplier) does not permit complete discrimination against the low energy neutrons, the differential cross-sections are not the elastic cross-sections. However, a crude estimate indicates that the low energy neutrons do not contribute more than 20 per cent to the measured differential cross-sections. The cross-sections exhibit strong maxima in the forward direction. Iron and lead have secondary maxima at 110° and 90°, respectively. The total cross-sections were also measured at 3.7 Mev by a standard transmission experiment giving 2.55, 3.51, and 7.60 barns for Al, Fe, and Pb, respectively, in good agreement with the values of Nereson and Darden.[1] The difference between the total cross-section and integral of our differential cross-section gives a measure of the total inelastic cross-section.

THE SCATTERING OF FAST NEUTRONS BY IRON, LEAD, AND CHROMIUM*

BY

M. A. ROTHMAN, D. W. KENT, JR. AND C. E. MANDEVILLE

Fast neutrons of 3.7 Mev energy have been inelastically scattered in Fe, Pb, and Cr. Using a NaI-Tl crystal, a photomultiplier, and pulse-height selection, gamma-rays are detected from iron at 0.88 Mev, from

* Assisted by the joint program of the Office of Naval Research and the Atomic Energy Commission.

[1] N. NERESON AND S. DARDEN, *Phys. Rev.*, **89**, 775 (1953).

lead at 2.65 Mev, and from chromium at 1.43 Mev. The neutron energy spectrum obtained, using 4.2 Mev incident neutrons, a chromium scatterer, and photographic emulsion detection, shows two resolved groups: the elastic group and another, inelastic, corresponding to a Q of about -1.4 Mev.

RADIATIONS FROM As^{77} AND Ge^{71}*

BY

B. L. SARAF, J. VARMA AND C. E. MANDEVILLE

As^{77} was grown from its germanium parent when GeO_2 was irradiated by slow neutrons in the Oak Ridge pile. Gamma-rays decaying with the 40-hr. period were detected at 85 kev, 160 kev, 250 kev, 270 kev, and 520 kev, the relative intensities of the unconverted quantum radiations being 3, 1, 20, 0.04, and 6. Gamma-rays of lower energies may also be present. The continuous nuclear gamma-ray spectrum accompanying orbital electron capture in the 11-day Ge^{71} has an end point at 225 ± 12 kev, yielding a value of 4.3 for log ft of the allowed electron capture transition which may be designated as $p_{1/2} \rightarrow p_{3/2}$, in agreement with the assignment of a spin of $\frac{1}{2}$ to Ge^{71} in accord with the shell model.

THE THEORY OF SECONDARY EMISSION†

BY

J. F. MARSHALL

The theory of the production of secondaries inside a solid has been investigated and it is found that the expression for the internal secondary yield is identical in form to the corresponding expression for the ionization of isolated atoms. (A similar result has been obtained by Baroody.[1]) This result has been used in the analysis of the energy dependence of external secondary emission, and, if the influence of primary scattering is included, a result is obtained which is in fair agreement with experiment.

* Assisted by the joint program of the Office of Naval Research and the Atomic Energy Commission.

† Assisted by the Office of Naval Research.

[1] E. M. BAROODY, *Phys. Rev.*, **89**, 910 (1953).

LINEAR ELECTRON ACCELERATOR*

BY

J. F. MARSHALL AND R. A. SHATAS

Several important improvements have been made in the linear accelerator in the past year.

1. The stability of operation has been greatly improved by water cooling the cavities and magnetron, thereby practically eliminating warm-up time and the necessity for frequent adjustment.
2. The output current has been greatly increased by redesigning the injection system, and peak pulse currents as high as 50 milliamperes can now be obtained, whereas only 1 or 2 milliamperes were obtainable previously.
3. A remote control system has been constructed and the machine can now be operated entirely from the remote station.

A second accelerator is under construction which is designed to produce electron energies of from 3 to 4 Mev, as compared with the 1.3 Mev output of the present machine. It is expected that the present high output currents will also be obtainable with the larger accelerator, since a study of the beam characteristics shows that the electrons are strongly focused by the cavity fields and that practically the entire current loss occurs in the first cavity.

* Assisted by the Office of Naval Research and the Office of Ordnance Research.

NOTES FROM THE FRANKLIN INSTITUTE LABORATORIES FOR RESEARCH AND DEVELOPMENT

NICOL H. SMITH, DIRECTOR

GYROSCOPE RESEARCH AT THE FRANKLIN INSTITUTE

BY

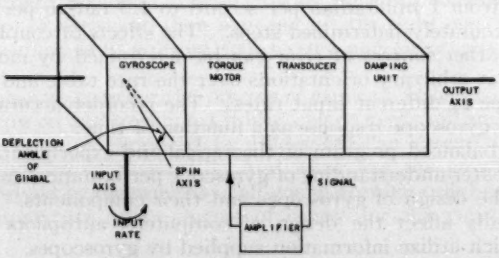
HOWARD H. BROWN

The Franklin Institute Laboratories have, during the past twelve years, engaged in numerous research and development projects in the field of gyroscopes. These have included research studies on eddy-current constrained gyroscopes, the development of rate and attitude gyroscopes for aircraft fire control systems, and others.

About one year ago, the Electrical Engineering Division undertook a program of research on angular rate gyroscopes, with the aim of uncovering non-linearities in the behavior of such systems and their effects upon the accuracy of angular rate information supplied by the gyroscopes. Gyroscope systems for measuring angular rates are widely used as sensing devices for airborne fire-control computing systems and in connection with autopilots.

The gyroscope operates according to the principles of rigid body dynamics. The heart of a gyroscope is the spinning wheel or rotor. Associated with the spinning rotor is a momentum vector having the direction of the axis of spin. Any change in the direction or magnitude of this momentum requires the application of a torque equal to the time derivative of the momentum.

In practice, the speed of the rotor is held constant, thus fixing the magnitude of the momentum. Changes in momentum can then occur only in its direction. If a torque with component perpendicular to the spin axis is applied to a free gyroscope, precession results. The necessity for high-accuracy free gyros has resulted in the development of gyros of high angular momentum and very low precession rate. These have wide application in maintaining a stable reference position in inertial guidance systems, in measuring attitude angles, and in controlling guided missiles.



Schematic diagram of rate gyro.

In single-degree-of-freedom angular rate gyroscopes the gimbal which supports the gyro rotor is constrained to move, against suitable restraining torques, about an axis perpendicular to the spin axis, the output axis of the gyroscope. If the case supporting the gimbal is given an angular velocity about an axis perpendicular to the spin axis and the output axis, a reactive torque around the output axis causes the gimbal to deflect. This reactive torque is equal to the product of the gyro's angular momentum and the input turning rate.

The figure is a schematic diagram of a single-degree-of-freedom angular rate gyroscope. The angular deflection of the gimbal is converted by a transducer into an output voltage or signal. This signal is used through a feedback amplifier to develop in a torque motor a torque in opposition to the acting torque of the gyro. The angular deflection of the gimbal is thus continuously being minimized. The current fed to the torque motor is used as a measure of the input angular rate. The response of such a rate gyroscope may be examined mathematically by the methods of servo-mechanism theory, and servo techniques may be used to improve the properties of the feedback loop.

Ideally, a single-degree-of-freedom angular rate gyroscope should respond only to the component of angular velocity of the gimbal support about the input axis. However, when the gimbal of a constrained gyroscope is in the deflected position, the angular velocity component of the gimbal around the spin reference axis of the gyro produces an additional torque around the output axis. This effect is called coupling. When the gimbal's angular velocity around the spin reference axis is a function of time, and in applications this is generally the case, interesting problems in non-linear mechanics result. Both Hill's equation and Mathieu's equation are exhibited in the equation of motion of such a system. Coupling effect produces error in the output of a rate gyroscope; the coupling effect is minimized by keeping the maximum angular deflection of the gimbal small. The mathematical treatment of coupling effect, non-linear damping, and time-varying gyroscope parameters leads to a new field of applications of non-linear mechanics.

A rate table and a special, high-speed oscillographic recorder are used for testing gyroscopes. The turntable provides angular rates in the range from 1 milliradian per second to 1.3 radian per second, in fourteen accurately determined steps. The effects of coupling, unbalance, and other sources of error can be determined by mounting the gyroscope at arbitrary orientations over the rate table and examining the response to different input rates. The recorder accurately prints on film the gyroscope response as a function of time.

A well balanced program of theoretical and experimental research leads to greater understanding of gyroscope performance, and improvements in the design of gyroscopes and their components. These end results vitally affect the design of computers, autopilots and other devices which utilize information supplied by gyroscopes.

NOTES FROM THE BIOCHEMICAL RESEARCH FOUNDATION

ELLICE McDONALD, DIRECTOR

THE FUTURE OF CANCER RESEARCH

BY

ELLICE McDONALD, M.D.

To forecast development in the future is a thankless job—for who can tell? I was once asked to write a prophecy for placement in a corner stone as to the state of biological investigation fifty years in the future. I refused, saying that if I could know what it was to be in two years I would be satisfied.

My only excuse is that I have now been at the job of cancer research for 52 years, interrupted it is true by a couple of wars and other interludes. Still, that time included a lot of experiences, particularly as three-fifths of it included directing others in cancer research. Looking back on this time I wonder how slow was the progress of thought and experiment. During this time, I followed many leads: Fibiger's work (1) in 1927 on experimental production of transplantable cancer; the colloidal aspects of the vital system; tissue culture or the growing of cells for which we transplanted a pupil of Fischer's from the Kaiser Wilhelm Institute in Berlin; enzyme studies in 1931 (2) for which we transplanted a complete enzyme laboratory from the old German University in Prague, headed by a pupil of Willstaetter, for few in the United States knew anything about enzymes and certainly none about enzymes in cancer. All this was without any real leads to the future of cancer research. I began to think that I was accumulating a lot of isolated uncorrelated facts about cancer and finding that the research was upon an experiment-and-error basis and not on scientific method. As a matter of fact it still is on the experiment-and-error basis as may be seen from the late meeting of the Association for Cancer Research at Chicago with its hundreds of papers. Experiment-and-error type of work is all right if enough experiments are done but sometimes it takes an awful lot of them and then may not hit the right one, whereas the scientific method is to go from step to step and eventually arrive at a solution.

So I began to consider the process of cancer. It had to have something to grow on and to support it like a plant needs soil and fertilizer—a growth factor as expressed by Fildes (3) "an essential metabolite which cannot be synthesized." And what was the growth factor in cancer: it would be the same as in all young growing cells because cancer cells are immature cells—glucose of course for the main source but

maybe other things, as different types of bacteria require special types of metabolites. *Escherichia coli*, for example, requires ammonium. But cells in tissue culture require glucose, so glucose anyway—maybe something else, but glucose anyway.

And if all chemical processes in vital systems are enzymatic in character: in the growth of cancer probably many enzymes are involved, not only one. And as glucose is at least the chief metabolite it would be the substrate and the enzymes could be called the glycolytic enzymes. Hexokinase is undoubtedly one but there are at least two types of hexokinase. So for now we can group the enzymes as the glycolytic.

So to get an enzyme inhibitor—but that wasn't so easy because there was nothing to go on and possibly there were several enzymes. After all, for years men have sought for a single substance which would be a cure for cancer, a thought akin to a belief in magic. But if there was any rationale for the wish, it would rest upon the conditions that there was one enzyme involved in cancer cell division and that it was possible to find a specific inhibitor. But there is no known chemotherapeutic agent that acts solely by blocking a single enzyme system: the action is invariably multiple so an inhibitor for cancer enzyme would block other systems.

But there are two factors in enzymatic reaction—the enzyme and the substrate. And in this case glucose is the substrate. Inhibition can be obtained also by substrate displacement by *chemical analogues* which are not in themselves metabolized. So has grown up the theory of biological antagonism dependent upon structural analogy and competitive inhibition. Chemotherapy in its broadest aspect is the reflexion of substrate displacement and enzyme inactivation (4).

So we searched for a chemical analogue of glucose and found one in 2-desoxy-D-glucose (2DG) the structure of which is identical to that of ordinary D-glucose with the exception that the hydroxyl group on the second carbon atom has been replaced by hydrogen. Almost all living cells possess enzyme systems capable of using glucose as a substrate for chemical transformations which provide energy for vital processes (5). The structure of 2DG maintains the identity of those atomic configurations in glucose believed to be involved in the attachment of some enzymes and it was anticipated that the competitive inhibition of glucose-using enzymes by 2DG, would occur.

This was shown by the study of the effect of 2DG on the anaerobic metabolism of yeast since this material is capable of carrying out glycolysis which in its initial stages at least is almost identical with that used by mammalian tissues. The study was made by Gladys E. Woodward of our laboratories by means of the Warburg manometer methods and published in the JOURNAL OF THE FRANKLIN INSTITUTE, April, 1952 (6). 2-Desoxy-D-glucose (2DG) was found "to inhibit strongly the anaerobic fermentation of glucose by living yeast" (G. E. Woodward, 6)

by competitive inhibition. This result was found to be consistent and so in cancer slices and cancer homogenates.

Not wanting to plump for 2DG without trying others, Miss Woodward studied the glycolytic inhibiting qualities of a number of other materials. The table shows the degree of inhibition.

Glucose Analogs Tested for Inhibition of Yeast Fermentation

1:1 With glucose	Inhibition Anaerobically, %	Aerobically, %
1. 2-Desoxy-D-glucose	76–80	75–82
2. D-Arabinose	0	0
3. D-Xylose	0	0
4. D-Sorbitol	0	0
5. Polygalitol	0	0
6. *iso*-Rhamnose	0	0
7. 3-Methyl-D-glucose	0	0
8. D-Glucosamine	41–49	0
9. Phenyl-β-D-glucothioside	0	not tested
10. 2,3,-Anhydro-α-methyl-D-alloside	0	not tested
11. N-Acetyl-D-glucosamine	0	0
12. D-Glucosone	56	88
13. 3-Desoxy-D-glucose	0	not tested
14. D-Glucononitrile	0	0
15. 2-Desoxy-D-allose	0	not tested
16. L-Sorbose	0	0
17. 2-Chloro-2-desoxy-D-glucose	12	not tested
18. D-Gluconamide	0	0
19. 2-Desoxy-D-gluconic acid	0	0
20. D-Ribose	0	not tested
21. 2-Desoxy-D-ribose	0	0
22. 2-Desoxy-D-xylose	0	0

This table shows the glycolytic inhibition but after all that is not the whole story, although it may be the most important part of it. There is the mechanism of cancer cell mitosis. So there are two factors, glycolysis and mitosis. We set up the study of mitosis by means of tissue culture which for its usual medium requires 1 g. of glucose to 1000 cc. So it was possible by substituting 2DG for the glucose to find the effects. The mitotic index was used rather than the growth of the culture because that is difficult to measure accurately and the mitotic index can be upon a quantitative basis.

The mitotic index with 2DG substituted for glucose was found much decreased by Ely, Tull and Hard (6). "The effect of 2DG, obviously, was inhibition of growth rather than killing since slight growth occurred and a low mitotic index was found . . . and cultures later grew at an apparently normal rate, after glucose was substituted in the medium for 2DG."

So 2DG is not the whole story. It inhibits while it is applied but when it is not there, the cells begin to grow again. Still, glycolysis is the most important factor in cancer growth and division and none of us in this laboratory can visualize control of cancer without inhibition of the glycolytic metabolism.

We set two test practices:

1. The glycolytic inhibition studies in yeast by means of the Warburg manometers; and
2. The mitotic index in tissue culture.

One measures the metabolism: the other the mitosis in tissue culture. So far these results have run consistently together. It is possible they may serve to screen substances likely to be beneficial in cancer therapeutics. Time will tell their true value, but at least these tests give a valuation which can be quantitatively arrived at where there was none before and their value will be finally measured by further experiments.

Glucosamine was studied by us but put aside as we found 2DG five times as great as an inhibitor of glycolysis. But Quastel and Cantero (7) published a paper in February 7, 1953 *Nature* ten months after our first paper (6), on the lengthening duration of life of certain cancer animals under glucosamine treatment and found the treatment increased the duration of life although all finally died.

Glucosamine is similar to 2DG except that there is an amine on the second carbon of glucose instead of a hydrogen, so it is also a chemical analogue.

This excellent paper of Quastel and Cantero led us to review our work upon glucosamine and give this substance more thoughtful attention. The results have been published in the June 1953 number of the JOURNAL OF THE FRANKLIN INSTITUTE and confirm the fact that D-glucosamine is a glycolytic inhibitor of glucose metabolism and a reducer of the mitotic index in tissue culture but not in either of these phases as influential as 2DG. The fact that it did not cure the cancer but merely prolonged life, put it in the same category as 2DG which in animal experiments showed occasional regression but more consistent termination of embryonic development in rats. The trouble with 2DG is that it is difficult to keep it in the body: it is precipitated out by the kidneys and brings with it a lot of glucose. If we could keep it in sufficient quantities we could hope for better effects. Maybe we will find some way to do it.

But what has been demonstrated is that the intact cancer cell exhibits an unusual ability to utilize glucose by the process of anaerobic glycolysis with the elaboration of lactic acid (8). And the admission of oxygen to such a glycolyzing system fails to interrupt the glycolytic process (negative Pasteur effect) a phenomenon foreign to almost all mammalian tissues. The facts suggest that the anaerobic type of

glycolysis is the most important aspect of metabolism in neoplastic tissue. Yeast cells have been shown to have a most similar type of anaerobic metabolism and in our belief may be used as a test material for glycolytic inhibition and similar consistent results have been shown with cancer slices and cancer homogenates as with yeast as a test material.

But this is not the whole story. There remain the nucleic acids which may be regarded as carbohydrate in nature. Both ribo- and desoxyribo nucleic acids are essentially long-chain polymers of N-glucosides of ribose and desoxyribose bound together by phosphate ester linkages. DNA is one of the characteristic substances of the genes whose aberrations must be responsible for neoplastic proliferation. A specific antagonist for desoxyribose or DNA might produce spectacular changes in dividing cells. That is a field where we are now working. If it is found, it may be additive to 2DG although glucosamine was not found to be, probably because each was competing for the same substrate.

This statement has been written with the hope of interesting others to extend the work, to test the validity of our test materials and if possible to place cancer research upon a scientific basis rather than the experiment-and-error accumulation of isolated uncorrelated facts as it exists at present. We wish to stress the importance of glycolysis of glucose in the metabolism of cancer as there has been little attention paid to it as an energy requirement in cancer metabolism and cell division. We invite others to give it their attention and we realize that it is only an important part, not the whole story. But the work of Quastel and ourselves shows that there is a lead here.

What has been stressed in our laboratories is the importance of the anaerobic glycolytic metabolism and the possibility of its inhibition by means of a chemical analogue to glucose which is not itself metabolized. The effect of chemical analogues in the inhibition of the metabolism of growth substances of bacteria have been increasingly studied since 1940 but this seems to be the first application of the theory to cancer growth.

Of course, it is only a part, an important part of the story: the rest lies in the permanent control of cell division.

But at least we have removed cancer research from the experiment-and-error basis to an approach to the scientific method. To aid in this we have set up two test methods to screen substances likely to control cancer: (1) The measurement of glycolytic inhibition by means of the Warburg manometer method, and (2) The mitotic index measurement by means of tissue culture. These have given consistent results in our laboratory both for yeast and for cancer cells and cancer homogenates. We hope that their promise will be fulfilled and in this laboratory we find it difficult to imagine cancer control without control of the glycolytic metabolism of the cancer cell.

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BOOK REVIEWS

THE ROAD TO ABUNDANCE, by Jacob Rosin and Max Eastman. 166 pages, 15 × 21 cm. New York, McGraw-Hill Book Co., Inc., 1953. Price, $3.50.

From time to time our journals convey gloomy forecasts of the impending exhaustion of our natural resources. Those who have tried to take an optimistic view of the future have never succeeded in sounding a note of conviction. They have been hopeful rather than convincing. However, here we are presented with the ultimate in conviction. A research chemist and an able dialectician have joined forces to prove that chemistry can solve all the material woes of humanity. In a breathless rush from one laboratory experiment to another, the authors explain how chemists can infallibly rescue mankind from its age-old bondage to conventional materials. We can be liberated from dependence upon plants for food and fiber and from the back-breaking labor of mining minerals. By manufacturing foods in factories instead of growing crops, we are promised a vast saving of productive floor space as well as a conservation of water and an efficiently balanced production that will be independent of a capricious climate.

The chemist, too, can free us from the tyranny of the mine. The knowledge of the correct utilization of solar heat will liberate the world from the need for coal and oil. The sea will be induced to yield untold mineral wealth.

As described in *The Road to Abundance*, this is no pipe-dream. It is hard fact reeled off without effort and without so much as a pause to allow doubts to enter. The case is presented without documentation, the book being innocent of footnotes, bibliography, or even index. But there is no doubt whatever that the authors have a complete mastery of their subject. The book is compact with physico-chemical knowledge. The one defect (and it is grave) is the complete disregard for any distinction between what is highly probable and that which is only remotely possible. A thing has only to have been done in a test-tube to be capable of reproduction upon a universal scale.

Max Eastman has had too many tussles with economic problems to be unaware of the immense problems involved in this essay upon the complete "chemistic" society, but they are all happily dismissed in a phrase. Freedom from plants and the mine can be achieved at will by organizing, mobilizing, and financing with the efficiency of a Manhattan Project on a large scale. It may require the organization of several projects that would dwarf the Manhattan Project but the authors think the effort is not only worth-while but necessary. It will be interesting to have the economists' views upon the by-product effects upon our civilization of this promised "chemistic" age and we would enjoy a gourmet's opinion upon sodium glutamate flavored Chlorella as a substitute for *filet mignon*, but these are doubtless only deferred until the "chemistic" age is nearer to being forced upon us.

Meanwhile, the book is highly recommended as an exciting and readable account of what the chemist has already accomplished and what his current work foreshadows. Its chemistry and physics are sound and thorough, if its optimism is a trifle overdone. The book can be safely prescribed as an antidote for depressed thought on the subject of the exhaustion of natural resources. T. C.

BETWEEN THE TIDES, by Philip Street. 175 pages, illustrations, 14 × 22 cm. New York, Philosophical Library, 1953. Price, $4.75.

Those of us who visit the seashore for our vacation take too much for granted. We see various types of plant and animal life and never really learn what is going on about us. The principal reason is that there is nobody we can turn to for an answer to the questions which arise. As a result we live in a separate world.

However, today there is a bridge for these two worlds. Now in a delightfully, fascinating book, *Between the Tides* by Philip Street, we are introduced to this old world we never

knew. The author introduces his book with a little understood point. The life on the seashore is dependent on the tides and all the animals found there are governed by the range of the tides through the day and the month.

He shows that there exists a constant struggle to maintain life in spite of the stormy seas, the receding tides and the presence of enemies. These problems have in the years modified the structures of the animals so they can cope with the difficulties.

In a book of sixteen chapters the author has skillfully selected his materials so that the reader can use it as a guide. He begins by describing the plant life—the weeds which are so much in evidence especially after stormy weather and then continues through the crustaceans, the worms, the sponge, fish and ends with the sea pasture—the diatoms and the plankton which provide food for the world of the sea.

The author quotes from naturalists of the past who had the happy faculty of describing events with almost a camera eye. One most intriguing description is of the attempt to capture a starfish in the accepted manner of plunging it into a bucket of cold fresh water and the insistence of the starfish of committing suicide before it could gracefully die intact as a perfect specimen.

The book is skillfully written, admirably illustrated and will make a pleasant companion on the next visit to the seashore.

I. M. LEVITT

CHEMICAL CONSTITUTION, by J. A. A. Ketelaar. 398 pages, diagrams, 16 × 23 cm. Houston, Elsevier Press, Inc., 1953. Price, $6.50.

The subject matter of *Chemical Constitution* by J. A. A. Ketelaar is explained by its subtitle, "An Introduction to the Theory of the Chemical Bond," although the word "introduction" seems somewhat out of place. This book is an English translation (by L. C. Jackson) of the 1952 edition in Dutch (the 2nd Dutch edition). There are just five chapters to the book. The first is a brief discussion of the periodic system and the four types of bonding—the ionic bond, the atomic bond, the metallic bond, and van der Walls bonding—each of which is the subject of one of the four remaining chapters.

As mentioned, above, the word "introduction" hardly seems to be applicable here since the elementary facts of chemical bonding are usually studied before certain of the ideas needed for an understanding of this book are encountered. It would seem that this text would most profitably be used by those who already have a background in some more advanced topics, particularly wave mechanics, and also a knowledge of such basic ideas as hyperconjugation, resonance, hybridization, and even of the four types of bonding which are nowhere explained in the usual organic textbook sense or specifically defined.

Bearing these things in mind, *Chemical Constitution* gives a complete discussion of the theory of the chemical bond with extensive examples on each topic discussed. For each type of bonding, the theory is first explained and then the consequences of the theory are considered. Small print has been used extensively for parts which the author thought could be omitted by the reader.

Specifically, it should be mentioned that there is an extensive treatment of wave-mechanical calculations and that the discussions on a number of topics are very complete, logical, and informative. Included among these may be those on hybridization, valence states, complex compounds, and other topics.

In general, this book is certainly to be recommended for those having some background and a definite interest in this theoretical but forward-looking subject. No other text so thoroughly covers this material. The author, in discussing not only his viewpoints but those of others, which in some cases are considerably different, gives a well-rounded advanced picture of the field.

It appears that the book suffers in translation, at least in spots. Some of the material which should be easily understood seems to be phrased in such a way that it is difficult to read.

MARLYN H. BORTNER

ELEMENTS OF PROPELLER AND HELICOPTER AERODYNAMICS, by Daniel O. Dommasch. 178 pages, illustrations, 16 × 24 cm. New York, Pitman Publishing Corp., 1953. Price, $4.50.

Rotating airfoils designed for propulsive purposes and those designed for lift and propulsion both (that is, helicopters) can be treated similarly in many fundamental and theoretical aspects. With the trend today toward specialization, the student and graduate alike have little opportunity to broaden

their narrow perspective. Consequently, this little text on propeller and helicopter aerodynamic theory is quite timely, especially since there now remains no doubt as to the importance and future of the helicopter. The student and practicing engineer are provided with the means by which they may readily extend well-known propeller principles, and thus acquaint themselves with the fundamentals of helicopter theory.

The book is written on a level suitable for a one-semester undergraduate course. It had as its origin a series of lecture notes prepared by the author for a course given at Iowa State College. These notes were revised later for use at the Test Pilot Training Division of the Naval Air Test Center.

The text is divided into six major chapters. Chapter one contains a concise introduction to the basic principles and definitions. Chapter two deals with the blade-element theories, including compressibility effects. This is followed by a short chapter on propeller characteristics including consideration of the propeller as a whole, dual propellers, and effects of pitch and yaw. The fourth chapter discusses axial flow through the helicopter rotor during vertical ascent and descent, hovering flight and autorotation. Chapter 5 covers the forward flight aspects of helicopters, and the concluding chapter 6 provides the reader with a fine background in the treatment of helicopter stability and control.

Illustrative examples, the many charts, illustrations and references contribute significantly to the general usefulness of the book.

SAMUEL M. BERKOWITZ

ION EXCHANGERS IN ANALYTICAL CHEMISTRY, by O. Samuelson. 291 pages, diagrams, 16 × 24 cm. New York, John Wiley & Sons, Inc., 1953. Price, $6.50.

The most recent developments in techniques of analytical chemistry have been in the use of ion-exchange resins. These materials provide a simple, rapid, and accurate means for performing chemical analyses. They also offer new and better methods for purifying and recovering valuable chemical compounds. As with many other modern technologies, there has been a great deal of research and practical application work published during the last twenty years scattered through scientific journals in many countries. The author uses this fact as his reason for writing the book. The book presents a detailed discussion of the principles involved in the use of ion-exchange resins, but is not intended as a complete reference book for specific analytical procedures. It also provides a complete and convenient source of reference to early or classical work together with the most recent thinking by the experts in the field.

The book is divided into four parts as follows: (1) An introduction consisting of a resumé of the classical work of the field and the scope of ion-exchange analyses and chromatography. (2) A section on the fundamental properties of these resins including adsorption equilibria and kinetics. The principles of operating ion-exchange chromatographic columns is also to be found in this section. (3) A practical part including choice of resin, optimum particle size, and the techniques involved in both analytical procedures and chromatographic adsorption. (3) An application section with suggested uses in organic, inorganic, and physiological chemical research problems. Some references are also made to the more important industrial uses.

The ion-exchange resins have provided the analytical chemist with a new and valuable research and development tool. Dr. Samuelson has written this book with the purpose of stimulating these chemists to more generally accept and to use the ion-exchange techniques in their work. He has put into one volume all of the early or classical work together with the most recent advances in the field. He makes some provocative suggestions as to further uses for these resins and also offers considerable guidance to workers who wish to use them for the first time. Obviously this book belongs in the hands of every analytical chemist in either research or development. It is the opinion of the reviewer that this book would also serve as an excellent supplement to any advanced course in analytical chemistry.

D. H. RUSSELL

ELECTRO-MAGNETIC MACHINES, by R. Langlois-Berthelot (translated by H. M. Clarke). 535 pages, diagrams, 14 × 22 cm. London, Macdonald & Co., Ltd., 1953. Price, 65s.

Translated and revised from its original French version, Langlois-Berthelot's volume on electrical power engineering brings to the profession something quite different from most American and English textbooks. The

differences are strikingly apparent when comparison is made with other recent texts such as *A–C Machinery* by Bailey and Gault or *A–C Power Systems* by Clarke. The results are both good and bad; for one is pleasantly surprised to find numerous convenient handbook items such as methods for measuring temperature-rise, but disappointed over the omission of certain modern developments such as synchros and control circuits.

The chapters of most books on rotating machinery are usually organized so that the various machine types are taken up individually and separate consideration is given to each class. Berthelot's book does not follow this convention, but, alternately, discusses numerous general topics in thirty chapters with a great many asides worked in as subdivisions. This approach is very confusing and requires one to do a great deal of searching to find the information wanted on a given type of machine whose characteristics may be scattered through many chapters. Also, the value of a good index has been overlooked.

The first impression one receives in studying this book is that its author tried to convey a lifetime of broad industrial experience. The volume contains numerous interesting but irrelevant discussions, including much general philosophy which might be criticized as padding were it not for the sincerity with which it is given.

Some divisions of the book result from the author's attempt to view the subject consecutively from the standpoint of the designer, then from the standpoint of the user. Bringing in these various aspects makes for a very broad coverage of the material and means that only brief mention of most topics is possible. This is one reason for the comment that the book is fairly descriptive and does not provide the working details required for practice; nor is the treatment sufficiently basic to be called a theoretical contribution.

C. W. HARGENS

ADVANCES IN CANCER RESEARCH, Vol. I, edited by Jesse R. Greenstein and Alexander Haddow. 590 pages, 16 × 24 cm. New York, Academic Press, Inc., 1953. Price, $12.00.

The accelerated advance of science towards an understanding of the physical and chemical nature of our world has been most impressive in the last few decades. The uncovering of so many interesting phenomena has made it easy for the scientist to embark on a satisfying course of action. Perhaps too many of us have been distracted, by these conquests, from the tremendous and fascinating challenge which confronts those who attempt to understand the fundamental mechanism of living matter itself. Nowhere is the urgency of this challenge brought to mind more forcibly than in the complex annals of "the gallant and dedicated quest for comprehension and mastery of (the) ancient and elusive disease"[1] of cancer.

The ten phases of research described in this book by authorities in the respective fields provide a remarkably lucid picture of the status of cancer research for the reader with a reasonable background in the physical sciences. Each section is forcefully and logically presented, and is similarly divided into several chapters which always include an introduction and final discussion. The presentations are essentially reviews, or documentary in nature, providing apparently complete references. But in practically all cases they are, to a reasonable extent, a function of the conclusions of the individual authors. The result is a form possessing many of the more interesting characteristics of an original paper.

This first volume illustrates the variety of philosophies employed in cancer research. Examples of this diversity are sections by C. A. Coulson on electronic configurations in carcinogenesis, L. Dmochowski on the milk agent in the origin of mammary tumors in mice, and A. Tannenbaum and H. Silverstone on nutrition in relation to cancer.

Perhaps one of the most laudable characteristics of the book, and one essentially common to all chapters, is the realistic attitude with which all the authors admit to the formidable task ahead. In the light of the facts now known and therein presented, it would seem that the final and absolute cure and prevention of cancer are strongly dependent on a reasonable understanding of living matter. Such an understanding should extend to all the chemical and physical properties of the cell itself. To this end, one might conclude that it would be well to increase significantly our efforts in fundamental or basic studies.

[1] Quotation from the Editor's Preface.

Such studies might be illustrated by C. Coulson's section, "Electron Configuration and Carcinogenesis" and C. Heidelberger's "Applications of Radioisotopes to Studies of Carcinogenesis and Tumor Metabolism." These increased efforts need not preclude the other studies which may provide more immediate means of relief and continue to save many lives.

As the editors state, it is their purpose "that this and succeeding volumes of this series shall reflect (the) steady and inevitable march of the tides of our knowledge and increasing understanding (of cancer)." This purpose seems to have been achieved with notable success in this first volume. It is to be fervently hoped that the volumes which begin to record the understanding necessary to solution will not be many volumes separated from the present one. ALBERT L. MYERSON

EXPERIMENTAL NUCLEAR PHYSICS, VOLUME I, edited by E. Segrè. 789 pages, diagrams, 16 × 24 cm. New York, John Wiley & Sons, Inc., 1953. Price, $15.00.

This is the first volume in a series of three which are intended to "bring the experimentalist up to date in experimental techniques, point out to him the significant facts and data, and indicate the broad lines of theoretical interpretation." The series is a cooperative effort of first rank nuclear scientists, each writing in the field of his specialty.

Volume I is a collection of five independent sections. Part 1—Detection Methods, by Hans H. Staub—discusses detector principles, associated circuitry and typical construction. Part 2—Passage of Radiations Through Matter, by Hans A. Bethe and Julius Ashkin—discusses passage of heavy particles, electrons and gamma rays, covering range energy relationships, ionization, scattering, radiation, photoelectric effects and pair production. In Part 3—Nuclear Moments and Statistics by Norman F. Ramsey—nuclear interaction with atomic fields, experimental methods and results are treated. Dr. Ramsey's second paper—Nuclear Two-Body Problems and Elements of Nuclear Structure—comprises Part 4 and discusses neutron and proton interaction, experiments concerned with these interactions, meson theory of the interaction and nucelar structure as found from atoms beyond the deuteron. In Part 5—Charged Particle Dynamics and Optics, Relative Isotopic Abundance of Elements, and Atomic Masses—K. T. Bainbridge discusses the mass spectrograph and its applications, as well as newer methods of measuring isotopic weights.

One of the great merits of this book is its emphasis upon fundamentals. It is definitely not experimental science in the cook-book style. In fact, Bethe and Ashkin do not discuss experimental methods at all. Each section follows a general plan: first, a brief introduction, then a thorough discussion of theory, then experimental techniques, and finally, applications and results. Results and theory are constantly measured against one another. It is to be regretted that Bethe and Ashkin omitted the introduction; a few generalizations about the material to follow is usually quite welcome. One of the nicer features is the historical introduction in Bainbridge's paper.

There is no question as to the value of the book—it will become a standard reference. There are many tables and graphs containing a great amount of collected data. Among the more extensive tables are ones on nuclear moments, isotopic weights, mass spectrometer doublets, nuclear reaction Q values and relative isotopic abundance. Each section has a lengthy bibliography and the book is well indexed. The publishers statement that "it fills a great need in analyzing and synthesizing all existing knowledge on nuclear phenomena" is correct, except that the word "all" should be toned down. LEONARD MULDAWER

RADIATION AND LIVING CELLS, by F. G. Spear. 222 pages, diagrams, plates, 14 × 19 cm. New York, John Wiley & Sons, Inc., 1953. Price, $3.50.

With the advent of the atomic bomb several generations of biological research have been compressed into comparatively few years. Within the past ten years we have seen the scale on which atomic energy has been used for destructive purposes and the world has no assurance that the events of Hiroshima and Nagasaki will not again be projected on the horizon. To counter the effects of another such catastrophe much work has been concentrated on the biological aspects of radiation.

While the beginning of this story dates back to Roentgen's discovery of radioactivity in 1895, the research pace has been highly accel-

erated with the advent of atomic energy and the availability of government sponsorship for research. While government research lends itself to a flood of literature there has been a distinct lack of contemporary books on the effects of radiation on the human body. "Radiation and Living Cells" is an attempt to correlate this many-faceted field in a single volume.

Dr. Spear designed his work to hold an intermediate position, somewhere between the popular level and the specialist. He has succeeded admirably in accomplishing his mission resulting in a book of authority and balance. As with many books of this type, it originated in a series of lectures and was then translated from the technical of the lecture to the popular of the general reader. The author has a happy faculty for clear, concise expression which makes the book palatable.

In the first two chapters the author sets the stage for the attack on the problem with the history of radiation biology and the description of the cell mechanism. Radiation of particular sections of the body are dealt with in subsequent chapters and finally the irradiation of the body as a whole. This is of utmost importance in the atomic age.

In the latter chapter he relates studies of the effects on body cells and tissues by radioisotopes. Their discussion centers around the use of isotopes as internal sources of radiation, the use of isotopes as tracers and the deleterious effects of isotope radiation. While this reviewer has seen this described many times in the past, Dr. Spear makes this exposition assume new significance.

The book concludes with a bibliography for general and scientific reading with the strictly medical books omitted. I. M. LEVITT

PHYSICS, by Noel C. Little. 648 pages, illustrations, diagrams, 18 × 24 cm. Boston, D. C. Heath & Co., 1953. Price, $6.00.

The publication of still another textbook on physics could not justify review in these pages were it not that this one departs from the conventional arrangement. This new course is organized around five fundamental concepts—length, time, force, electricity, and temperature—and five types of phenomena—energetics, flow phenomena, field phenomena, periodic phenomena, and quantum phenomena. The thirty-six chapters cover these ten units. The approach is based on underlying theory rather than on outward aspect, an approach which enables the author to present a much clearer interrelationship of the physical sciences. Algebra, geometry, and trigonometry are freely used, while those derivations involving use of the calculus are banished to an appendix. There is an abundance of problems.

Dr. Little and his colleagues at Bowdoin College have employed this organization of the course for more than twenty-five years with conspicuous success. It may be assumed, therefore, it fully answers those details which are important in teaching. The basic material is quite up-to-date, but the most interesting feature of the treatment is its new way of looking at old concepts. It may be unjust to single out any chapter for comment, but one called "The A B C of Relativity" found special favor with this reviewer.

Students who have to use this text-book will have every reason to congratulate themselves upon its choice. It is stimulating as well as instructive.

A word of praise is due the designer who has provided ample margins in a well printed book to accommodate an unusually large number of diagrams, leaving the text for the most part unbroken. Those margins will be ideal for making notes. T. C.

HANDBOOK OF STATISTICAL METHODS IN METEOROLOGY, by C. E. P. Brooks and N. Carruthers. 412 pages, diagrams, 16 × 25 cm. London, Her Majesty's Stationery Office, 1953. Price, 1£ 5/.

The study of meteorology involves the proper evaluation of a great deal of data from point sources. This involves the subjects of probability and statistics, especially when time is a factor in the analysis. Statistics has been used increasingly in this work, but this is the first book in English specially directed to the use of statistical methods in meteorology.

Most of the statistical principles used can be found in standard textbooks and derivations are not shown here. Where theorems are new and hitherto unpublished, proofs are given. The examples and exercises to be worked are almost entirely meteorological, and illustrate a wide range of problems which arise in meteorological and climatological work. Answers to the exercises are given at

the end of the book. Several advanced sections, marked with a star, are either more difficult or of less wide application than the unstarred sections.

One object of this book is to show how a series of observations can be analyzed to make the most use of it, and how the results of such analysis can be applied to answer specific questions about climate. Correlation and curve fitting are used to relate two or more series of observations which are not independent of each other such as temperature and humidity. A third section of this book concerns weather persistence from one observation to the next. Concerning periodicity, it is shown that the probability of an event is periodic rather than the event itself.

The authors carried out much of their original research during and after the war in England. They have submitted a bibliography of 162 references relating to their book, a large number of which are European and Asiatic works, and may be of interest to readers in the United States. The book should be possessed by every weather analyst.

A. D. HAY

BASIC MECHANICS OF FLUIDS, by Hunter Rouse and J. W. Howe. 245 pages, diagrams, 15 × 24 cm. New York, John Wiley & Sons, Inc., 1953. Price, $4.50.

The authors have prepared this text to provide material for a one semester course in Fluid Mechanics for junior engineering students. In following this aim, the principles of fluid statics are presented first. The significance of generalized variables—the Euler, Fraude and Reynolds numbers, in the treatment of acceleration, wave motion, and viscous flow—is pointed out. As noted by the authors, the use of dimensionless numbers represents one of the most simplifying features of fluid mechanics.

Next are treated the kinematics of fluids. The principle of continuity and the various characteristics of the flow pattern—velocity, acceleration, and separation—are discussed for typical boundary conditions. The use of flow nets in determining the flow pattern is illustrated.

Fluid dynamics is introduced thorugh pressure-velocity relationships. The energy and momentum principles are developed and applied to particular flow problems. Non-uniform motion of confined fluids is treated for several particular cases.

While the authors claim to emphasize general principles, nowhere is the basic one-dimensional equation of motion of a fluid presented in its entirety, although special aspects of this equation are used in the analysis of uniform and non-uniform flow in pipes and channels. The energy principle is stated only for the case of non-viscous and incompressible flow.

The elementary concepts of Prandtl's boundary layer theory are presented. Velocity distributions in laminar and turbulent boundary layers are discussed. These concepts and the Reynolds number are then used in a study of resistance to flow past immersed bodies of various shapes.

A final chapter on fluid compressibility examines the propagation of waves in fluids and analyzes the water hammer problem. The role of the Mach number as the criterion for similarity of elastic wave action between two geometrically similar states of flow is pointed out.

With the exception of the weakness in presentation of fluid dynamics, the authors have done an excellent job of presenting the engineering mechanics of fluids from a modern viewpoint.

HOWARD H. BROWN

STRUCTURAL PRINCIPLES AND DATA, issued by the Royal Aeronautical Society. 322 pages, diagrams, 16 × 26 cm. New York, Pitman Publishing Corp., 1952. Price, $8.50.

In an attempt to provide engineering data pertinent to the aeronautical field, a *Handbook of Aeronautics* was prepared some 22 years ago. Well received and very useful, subsequent single editions were issue in 1934 and 1937. A fourth edition, of which *Structural Principles and Data* is the first volume, is now in progress. Grown too large for a single edition, the handbook will be published in several volumes.

Divided into two parts, Structural Airworthiness and Structural Analysis, this first volume introduces the preliminary factors in the design of aircraft and treats of the various methods of structural analysis. Part I describes the various types of loads that an airplane encounters and that must be considered in the design. Such loads are symmetric

maneuvering, gust, tail-plane and elevator, aileron, landing gear, engine mounting and crash. In each case, the nature of the load and the development of the formula for its evaluation are presented. In Part II, the various methods of structural analyses applicable to aircraft are discussed. Such material includes stress-strain curves and physical properties of aluminum alloys, beams and struts, stability of plates and panels, flat and curved shear panels, wing stressing, and rings and curved beams. Many tables and charts expressing in graphical form the solution of the above structural analyses are included and add greatly to the value and usefulness of the book. A very extensive reference and bibliography concludes the book.

When the other volumes of the fourth edition of the *Handbook* are completed, one should have a very complete handbook on significant design data pertaining to all phases of aircraft design, if this first volume is any indication of what to expect.

E. W. HAMMER, JR.

THE HUMAN SENSES, by Frank A. Geldard. 365 pages, illustrations, 15 × 23 cm. New York, John Wiley & Sons, Inc., 1953. Price, $5.00.

Our present day technology has made the quantitative study of complex man-machine systems of ever increasing importance. Clearly, one prerequisite for the study of man's interractions with complex man-machine systems is a thorough knowledge of the physical stimuli to which man reacts and the manner in which these physical stimuli become transformed to experiences. Geldard has prepared a well written and fairly comprehensive discussion of the ways in which man detects changes in his environment. In particular, Geldard discusses vision, audition, skin senses, kinaesthesis, labyrinthine sensitivity, smell and taste.

The detail and sophistication of the coverage are adequate for an introduction to the psychophysiology of the senses. Depth of description has been sacrificed for ease of presentation. For example, Denham's top is discussed on p. 65 in a provocatively incomplete manner. The more than 300 references compensate in a fashion for the lack of exhaustive detail.

This book is valuable for getting oriented in the field of human senses, but a serious student or a technical man with a difficult problem will have to go to either the original sources or a more ambitious source book for his answers.

EZRA S. KRENDEL

DEMAND ANALYSIS. A STUDY IN ECONOMETRICS, by Herman Wold, in association with Lars Juréen. 358 pages, 16 × 24 cm. New York, John Wiley & Sons, Inc., 1953. Price, $7.00.

The laconic subtitle of this book is a fine example of an understatement. To be sure, its central theme is analysis of consumer demand, an important branch of econometrics. Yet, the subject is presented (by the principal author) in a conceptual framework of such latitude that this "study" is much more than a "monograph written in the dual form of a research report and a specialized textbook on econometrics." It is, in effect, a didactically masterful exposition of the emergent realistic scientific approach, in the spirit of "logical empiricism" (page 75), to observable phenomena, and of the prerequisite modern theoretical tools to whose development this author has made important contributions. Among these, the theory of stationary stochastic processes and, particularly, the theory and methods of regression analysis are presented in detail, respectively, in Parts III (pages 149 to 186) and IV (pages 187 to 252).

The introductory Part I (pages 1 to 80) and the above-mentioned Parts III and IV contain a wealth of ideas and many, in part, new mathematical results which will be valuable to any worker in the broad and expanding field of applied probability theory.

True to the spirit of "logical empiricism" which, as a matter of fact, permeates the entire book, the theory and methods of demand analysis are developed in Part II (pages 81 to 148) on the basis of the Paretoan axiomatic theory of consumer demand. This part forms the theoretical core of the book and is of primary interest to students and workers in the field of econometrics. Yet, the exposition of this theory will be perused with profit by competent readers whose interests lie outside this field. It offers an excellent concrete illustration of the power of modern methods of rational analysis when applied to complex and but partly deterministic processes which are too often popularly still believed to lie outside the pale of realistic rationalization.

The practical value of this theory is demonstrated by its very successful application in the evaluation of family budget data and market statistics collected by government agencies in Sweden. The findings of this extensive econometric investigation are presented in condensed form and discussed (both authors collaborating) in Part V (pages 253 to 322).

Throughout the book these empirical data and specific results are used to illustrate the quantities and relations contemplated by the theory and to provide realistic examples of various methods of data evaluation.

Numerous exercises appended to Parts II, III and IV are provided for the student. Text notes collected in an appendix (pages 323 to 336) contain valuable historic information and comments, as well as references to an extensive international bibliography (pages 337 to 346). A list of author references and a well organized subject index appear at the end.

A very excellent book, well suited to provide a thorough footing for the student of econometrics! The worker in the broad field of applied probability theory will welcome it as an authoritative source book on stationary time series and, particularly, on regression analysis. The intelligent general reader will find in Part I, Chapter 3 (pages 60 to 79) a lucid and not-too-technical exposition of the rationale and analytical tools of modern scientific approach to empirical problems.

O. R. SPIES

BOOK NOTES

INDEX (M.I.T. Radiation Laboratory Series), edited by Keith Henney. 160 pages, 16 × 24 cm. New York, McGraw-Hill Book Co., Inc., 1953. Price, $4.50.

With the publication of this invaluable index, the monumental M.I.T. Radiation Laboratory Series of twenty-seven separate monographs is finished. The index itself is a complete guide to the entire series and as such will be most useful. In addition, those who value the Radiation Laboratory Series will find more than an index to interest them, for the book contains a well-written foreword by Vannevar Bush, an eight-page account of the establishment of the Radiation Laboratory written by Karl Compton, a nine page story by Lee DuBridge of the organization of the Laboratory, and a six-page account of the trials of publishing the series, by the editor, Louis N. Ridenour.

TEXTILE FIBERS, YARNS AND FABRICS, by Ernest R. Kaswell. 552 pages, diagrams, 16 × 24 cm. New York, Reinhold Publishing Corp., 1953. Price, $11.00.

Since this is the first book to evaluate, on the basis of research results, the engineering, physical and chemical behavior of natural and synthetic textile fibers, yarns and fabrics, it will no doubt become a widely used reference. The book reflects the sponsorship of the Wool Bureau, since wool is used as a yardstick in evaluating the behavior of the other fibers and fabrics. Thoroughly covering the intrinsic properties of fibers and their relationships to the finished fabric in regard to abrasion resistance, crease retention, wrinkle resistance, air and water permeability, the book will serve not only textile chemists, physicists and engineers, but will be of use to people in the service, production and management departments of the various textile industries, as well as to those in such allied fields as plastics, paper, rubber, leather and clothing manufacturers.

A LABORATORY MANUAL OF EXPERIMENTS IN PHYSICS, by Leonard Rose Ingersoll, Miles Jay Martin and Theodore Alton Rouse. Sixth edition, 286 pages, diagrams, 16 × 24 cm. New York, McGraw-Hill Book Co., Inc., 1953. Price, $4.00.

A thorough revision of this comprehensive laboratory manual offers more material than a single course can cover, allowing the instructor a choice of seventy-seven experiments. Designed to meet requirements not only of physics majors, but also of nontechnical students, the manual also is broad enough to be used in any laboratory, regardless of what equipment is available. Thirteen out-of-date experiments have been withdrawn, seven new ones have been added and many others have been rewritten to include up-to-date material.

PHYSICS PRINCIPLES AND APPLICATIONS, by Henry Margenau, William W. Watson and C. G. Montgomery. Second edition, 814 pages, illustrations, 16 × 24 cm. New York, McGraw-Hill Book Co., Inc., 1953. Price, $7.50.

This new edition of a successful engineering physics text will be welcomed by all who have used the first edition (1949). Improvements include simplification of some of the more difficult sections and correction of some of the illustrations. To bring the book up-to-date, new material has been added, for example, the cosmotron, thermonuclear processes and fission, transistors, and the Schmidt camera. New figures, to illustrate the latest devices, have been added. The basic idea of the original text—stress on fundamental concepts and their applications—is maintained, and enhanced through the addition of many new worked out examples.

BEGINNING ALGEBRA FOR COLLEGE STUDENTS, by Lloyd L. Lowenstein. Second edition (first edition was in mimeograph form), 279 pages, 14×22 cm. New York, John Wiley & Sons, Inc., 1953. Price, $4.00.

Establishing Algebra as a logical structure from the start, this new textbook uses the student's experience in arithmetic to develop his understanding of the rules of algebra. Starting with the natural numbers and the basic rules of operation with them, the student is led to see the need for enlargement of this number system—first to zero and the signed integers, then to rational numbers, and finally the complex number system. This new text is geared to satisfy the needs of both an introductory mathematical course and a terminal course. Its main object is to make the student think for himself.

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CURRENT TOPICS

Hydrazine—A New Tonnage Chemical.—Hydrazine is one of the most versatile and intriguing materials to spring from the chemical laboratory. Based on ammonia, it is a highly reactive substance which first came into prominence near the end of World War II as one of the chemical fuels used in German rockets. It has since been produced in this country in limited quantities and has already found use in the manufacture of drugs, photographic chemicals, soldering fluxes, foam rubber, insecticides, plant growth regulators, metal-plating materials, boiler compounds, and other chemical products.

Completion in July by Mathieson Chemical Corporation of the first plant in the United States to produce hydrazine on a tonnage scale focuses new attention on its many potential applications in industry. Promises of important additions to the above list are seen in a number of the more than 2000 derivatives of the material that have thus far been reported, of which several score are under investigation in laboratories throughout the country.

In appearance, hydrazine is a clear, colorless liquid which closely resembles water. It has about the same density as water, and its boiling point is only a little higher. But there the resemblance ends. Hydrazine is one of the most highly reactive of the inorganic chemicals. It is also a very powerful and concentrated source of chemical energy.

The hydrazine molecule is comprised of two nitrogen atoms joined together, with two hydrogen atoms attached to each nitrogen. The hydrogens are readily replaced by other atoms or groups of atoms through a variety of chemical reactions, to provide an extremely wide range of possible molecular combinations and properties. The two nitrogens, however, are so tightly linked together that any reaction vigorous enough to pry them apart releases a large amount of energy.

Searching during World War II for a rocket fuel that would provide maximum energy per unit weight, the Germans worked with hydrazine hydrate (a form of hydrazine which contains about 36 per cent water) plus an oxidant such as hydrogen peroxide or liquid oxygen. Such a hydrazine-oxidant mixture was used in the powerful German rocket fighter planes, introduced near the end of the war, which were capable of climbing at the astounding rate of seven miles a minute.

During World War II hydrazine was made in the United States in limited quantities for experimental purposes and as a raw material for the manufacture of azides for shell detonators.

Immediately following the war Mathieson Chemical Corporation resumed a research program it had begun in 1939 on applications of hydrazine and inaugurated a companion program on hydrazine production under sponsorship of the United States Government. This coincided with substantial increases by the military services in their developmental activities on liquid-fuel rockets. In 1948 Mathieson began manufacture of hydrazine in pilot plant quantities at Niagara Falls, N. Y.

The Mathieson research led to two commercially feasible methods for

removing the chemically-combined water from the hydrate and carrying the material through to anhydrous hydrazine (95 per cent hydrazine). One of these methods involves further treatment of the hydrate by ammonolysis, while the other is based on a process known as azeotropic distillation.

Commercial-scale production of anhydrous hydrazine thus has been strictly an American development. The anhydrous form has major advantages over the hydrate for many uses. In rocket fuels, elimination of the 36 per cent water not only gets rid of useless dead weight but also does away with the need to dissipate part of the energy of the fuel in evaporation of water. As far as industrial uses are concerned, the anhydrous product significantly widens the range of possible reactions with other materials.

In July of this year Mathieson brought into production a new plant at Lake Charles, La., which for the first time provides the United States with a major source of hydrazine.

Initially the plant will produce not only hydrazine hydrate but also anhydrous hydrazine, monohydrazine sulfate, dihydrazine sulfate, hydrazine hydrobromide, and hydrazine hydrochloride. Other hydrazine derivatives will be made as demand warrants.

Military requirements are expected to take most of the plant's initial output. Indications are that non-military uses of hydrazine will develop further in the near future and are expected in time to become significant in terms of total demand.

Magnetic Drum Calculator—International Business Machines Corp. has announced the introduction of a new commercial electronic decimal calculator designed to meet the vast accounting and computing requirements in areas between those now served by its "giant brains" and the widely-used smaller machines such as the 604 Electronic Calculating Punch and the Card-programmed Calculator.

Called the IBM Magnetic Drum Calculator, the new machine combines one of the advanced memory devices and the stored program concept of IBM's big "701," recently announced, with new high speed reading capacity in the conventional punched card equipment to achieve a powerful data processing machine for commercial and engineering requirements.

In addition to its usefulness as an accounting and computing tool, the Magnetic Drum Calculator will be a vital factor in familiarizing business and industry with the stored program principles fundamental to electronic data processing equipment. Though its capacity is large, it is designed for exceptional ease of problem preparation and operation. A significant feature of this machine is its ability to check the accuracy of its answers.

Typical application of the Magnetic Drum Calculator is the simultaneous computation of insurance premiums and calculation of required reserves.

For the automobile industry, the calculator can "remember" several weeks' or months' production schedule of finished products, and at the same time calculate at high speed the number of parts required for their production. It can take into account such factors as "lead time" and indicate the dates on which orders must be placed for each part to be available for scheduled assembly.

Because of such advanced techniques as its large stored program and its extensive memory capacity, the drum calculator can incorporate in one machine operations in punched card accounting which once required several machines. A numeric decimal machine, it has up to 20,000 memory positions and can accept as many as

2000 individual operating instructions to facilitate commercial and scientific computations. It consists of three units: a magnetic drum unit with electronic calculating components, an input and output unit, and a converter. It occupies approximately the same area as two ordinary office desks and a filing cabinet.

The calculator also features a "Table Look-up" operation which facilitates the automatic searching of rate tables such as occur in the utilities, life and casualty insurance, transportation and other commercial fields.

The operation of these features as they might be used in a standard commercial application of the magnetic drum calculator is illustrated by following the course of events from the time a gas and electric meter has been read to the time the bill is prepared. While the utility company representative is peering with his flashlight into those out-of-the-way places where meters often are installed, the magnetic drum calculator, back in the home office, has stored on its drum the complete rate structure of the utility and is ready to consolidate a minimum of five operations into one processing. When all the individual meter cards are fed into the machine, the drum calculator, by means of its Table Look-up feature, selects the rate applicable for each class of service, makes all the necessary calculations for the bill amount, and punches the result into another card along with the indicative information necessary for bill preparation. In this application, internally contained self-checking features virtually eliminate the necessity for separate processing of the cards for checking purposes. The entire operation materially reduces the time involved in getting out the utility company's bills.

In addition to performing all the computations necessary for billing purposes, the calculator's remaining drum capacity also can be used for the distribution of revenue or rate statistics. This cumulative information may be kept on the drum as long as desired, regardless of the disposal of the data used in the individual meter billing operation.

By means of the console, the operator has control over all stages of the calculations and may manually insert instructions or data into any desired storage location, examine the contents of these locations, stop the calculation at any required point and begin calculation with any desired instruction located in the memory unit.

First Propulsion Equipment Installed for World's Largest Wind Tunnel.—Installation of the most powerful electric motor ever built—83,000-horsepower—completes the first major step in assembly of the 216,000-hp. electric drive for the U. S. Air Force's new transonic and supersonic wind tunnels. Completion of the two wind tunnels will permit the testing of full-size jet engines, guided missiles, and wing sections and fuselages of aircraft at speeds up to 2500 miles per hour.

The motor was installed at the Arnold Engineering Development Center, Tullahoma, Tenn. The Center houses important aeronautical research facilities, available not only to the Air Force, Army and Navy, but also to all parties—in government, industry, and science—interested in aircraft development.

The giant synchronous motor is one of two such units built by the Westinghouse Electric Corporation in East Pittsburgh, Pa. The second unit will be installed in the near future, along with two 25,000-hp. Westinghouse wound-rotor induction motors. Each 83,000-hp. motor stands 21½-ft. high, and weighs 225 tons. Their 122-ton rotors will turn at 600 rpm.

In addition to the 216,000-hp. four-motor drive, the rotating machine will have five compressors, now under construction at the Westinghouse plant in Sunnyvale, Calif. Work on these transonic and supersonic compressors is not expected to be completed for at least another year. The transonic compressor will be a single unit, but the supersonic compressor will be made up of four compressors coupled as one.

Blades for the compressors measure 2 ft. across the face, are 6 ft. long, and will be mounted on a spindle 18 ft. in diameter. Weighing almost two-thirds of a ton each, the blades will be solid forgings.

Although the machine will have the highest stored energy of any rotating mass ever built, it can be brought to a halt in about three minutes by using its wound-rotor motors as brakes, the energy being dissipated in liquid rheostats.

During operation, the wind tunnels will require 100,000 gallons of cooling water per minute to reduce the air discharge temperature to about 600 F. However, air in the test chamber will be about 100 F. below zero.

The transonic wind tunnel is scheduled for completion early in 1955. The supersonic tunnel will be completed in 1957.

Tester for Turbine Buckets.—A wailing, complaining inferno of a machine that puffs and blinks and blasts but doesn't move has been developed by scientists of the General Electric Research Laboratory to imitate a whirling turbine.

Designed to aid in predicting how modern gas turbine buckets will hold up after years of service, the raucous device simulates conditions present in the modern G-E turbine. Such buckets, shaped somewhat like small propeller blades, are mounted on turbine rotors to take the impact of hot gases blown against them to turn the rotor.

With an eerie howl and a beady, blinking light, the machine blows a scalding hurricane of gas through a slotted wheel against the bucket. Timed electronically to reach a critical resonance, the sound produced soon sets up a destructive vibration in the tapering bucket.

A winking blue stroboscopic light then "stops" this motion to reveal to scientists how much the bucket oscillates. Further details about strains placed on the metal are relayed to a battery of recording machines mounted on a nearby instrument panel.

Information leading to valuable predictions about metal performance of the buckets under stress can be determined "in a matter of minutes" by the machine, according to R. V. Klint, young G-E scientist who developed the bucket vibrator.

Klint explained that in his machine, resonance set up in the buckets soon causes sufficient vibration to produce fatigue failures in them. This testing, he said, can be done rapidly and at low cost.

Another important use of the machine, he reported, is in studying conditions that help buckets withstand vibratory forces. One such effect, called damping, is the restraint inherent in metals and bucket designs that prevents excessive vibrations during actual turbine operation. Rupture of the buckets could otherwise result from these vibrations, Klint pointed out.

"By using the laboratory machine, we are able to measure the magnitudes of these sources of damping and can test and develop various ideas which should put damping at the maximum for a given turbine design," the young scientist added.

Silver Graphite Sliding Contacts or Brushes.—An innovation in sliding contacts or brushes for servomechanisms, radar antenna operating units, calculating machines, midget motors and other exacting applications has been announced by the Stackpole Carbon Company, St. Marys, Pa. These Stackpole silver graphite units feature extremely low contact resistance and great resistance to welding for maximum contact efficiency and life at minimum cost. Lowest radio noise levels short of using more costly noble metals are obtained by using these silver-graphite units against a silver ring. For ordinary uses however, a copper ring or commutator will suffice.

Available in sizes from 1/16-in. diameter upward, Stackpole silver graphite contacts and brushes can be supplied with pure silver backing for easy spot welding or brazing directly to supporting arms or springs. They can also be furnished with copper backing, integral rivet, or other device to meet almost any mounting requirement. Units are supplied separately or factory-mounted to specifications. They are made of silver with almost any desired percentage of graphite. Standard grades range from 5 to 80 per cent graphite.

Electronic "Giant Brain" Has Memory Improved.—A new memory for the ENIAC (Electronic Numerical Integrator and Computer), first electronic "giant brain," was demonstrated early in July to Army Ordnance officials at the Philadelphia Research Center of Burroughs Corporation (formerly Burroughs Adding Machine Company).

The new static magnetic memory, as it is technically called, is the first memory device of its type ever designed and constructed. Using new magnetic techniques, the memory is one of the fastest in existence today.

Access to information is at the rate of 50,000 "words" per second—a word is a ten decimal digit number, for example, 9,378,462,571. The high rate of reading and storing information is made possible by the use of a new type of magnetic core, specially designed by Burroughs, and used for the first time in this memory. The same electronic and magnetic techniques used in the memory are under development at Burroughs' Philadelphia Research Center for application to information-processing equipment for business use.

Constructed under a commission from the United States Army Ordnance Department, the Burroughs memory will increase and quicken the ENIAC's operation, to give it the flexibility and capacity needed to solve large-scale problems. ENIAC's memory capacity will be increased six times by the addition of the Burroughs unit to its original small internal memory. The ENIAC will now be able to "memorize" 120 words at any particular instant.

ENIAC, oldest and most famous of the large-scale, high-speed electronic "giant brains," was built for the Army by the Moore School of Electrical Engineering, University of Pennsylvania, Philadelphia, during the last years of World War II. Dr. Irven Travis, now Burroughs Vice President in charge of Research and then Supervisor of Research at the Moore School, was associated with its construction.

Now housed at the Army's Ballistics Research Laboratories, Aberdeen Proving Ground, Maryland, the ENIAC is used to compute shell trajectories, firing and bombing tables, the problems of air flow around revolving missiles traveling faster than sound, and of atomic fission. The Burroughs memory will enable the ENIAC to compute these problems in less time.

Ferroelectric "Memory" Crystals.—Flat crystals, a few thousandths of an inch thick with the unique ability to remember vast amounts of information, have been developed by Bell Telephone Laboratories, the research and development organization of the Bell System.

One such ferroelectric crystal only half an inch square can store approximately 250 bits of memory for an indefinite period. Light amber-colored, these crystals are artificially grown from the chemical barium titanate.

Their creation came out of a communications need: memory is vital to the operation of the telephone system. A dial number, for example, is remembered on a set of relays in the central office until the connection is made with the other party.

Memory also figures prominently in computing machines, which employ electronic tubes, magnetic drums and other devices for this purpose. These generally occupy large volumes of space.

On the other hand, a few square inches of the crystals have a potential for information storage equal to many cubic feet of currently used apparatus. Hence the crystals may have profound significance in decreasing the space occupied by telephone switching systems.

Several members of the Laboratories' technical staff have cooperated in this development. The technique of using the crystals for memory devices has been developed by J. R. Anderson employing crystals grown by a special process. W. J. Merz has been carrying on studies on the relevant physical properties of these crystals.

The telephone dial system and most computers use special codes. Telegraphy and wireless telegraphy were made possible by the universal dot-and-dash Morse code. Even a player piano receives its musical impulses from a code punched on a paper roll.

Bell Laboratories' memory crystals store their information in the "binary" code. This code consists of only two symbols designated by either a "yes" or a "no." Words, sentences or a series of numbers can be coded by using a large number of these symbols just as the punched pattern in the player roll can represent a piece of music.

Coded information is "fed" into the crystals by the simple application of a plus or minus voltage, depending on whether a "yes" or "no" is desired. The information can be retained indefinitely in the crystal.

When the crystal is read out it simulates the human process of "bringing to mind." Other circuits then interpret the stored charges in a millionth of a second, using but microscopic amounts of electricity.

While no man-made storehouse for facts has ever approached the compact perfection of the human brain, these crystals are considered a significant step at least in miniaturization. It has been estimated, as a matter of interest, that it would require space the size of Grand Central Station to house the vacuum tubes, relays, condensers, wiring and associated equipment even to approximate imperfectly the capacity of the brain.